Boulder Valley School District
Excellence and Equity

M25
Mathematics
Curriculum Essentials Document

Boulder Valley School District
Department of Curriculum and Instruction
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Boulder Valley School District Mathematics – An Introduction to The Curriculum Essentials Document

Background

The 2009 Common Core State Standards (CCSS) have brought about a much needed move towards consistency in mathematics throughout the state and nation. In December 2010, the Colorado Academic Standards revisions for Mathematics were adopted by the State Board of Education. These standards aligned the previous state standards to the Common Core State Standards to form the Colorado Academic Standards (CAS). The CAS include additions or changes to the CCSS needed to meet state legislative requirements around Personal Financial Literacy.

The Colorado Academic Standards Grade Level Expectations (GLE) for math are being adopted in their entirety and without change in the PK-8 curriculum. This decision was made based on the thorough adherence by the state to the CCSS. These new standards are specific, robust and comprehensive. Additionally, the essential linkage between the standards and the proposed 2014 state assessment system, which may include interim, formative and summative assessments, is based specifically on these standards. The overwhelming opinion amongst the mathematics teachers, school and district level administration and district level mathematics coaches clearly indicated a desire to move to the CAS without creating a BVSD version through additions or changes.

The High School standards provided to us by the state did not delineate how courses should be created. Based on information regarding the upcoming assessment system, the expertise of our teachers and the writers of the CCSS, the decision was made to follow the recommendations in the ***Common Core State Standards for Mathematics- Appendix A: Designing High School Math Courses Based on the Common Core State Standards***. The writing teams took the High School CAS and carefully and thoughtfully divided them into courses for the creation of the 2012 BVSD Curriculum Essentials Documents (CED).

The Critical Foundations of the 2011 Standards

The expectations in these documents are based on mastery of the topics at specific grade levels with the understanding that the standards, themes and big ideas reoccur throughout PK-12 at varying degrees of difficulty, requiring different levels of mastery. The Standards are: 1) Number Sense, Properties, and Operations; 2) Patterns, Functions, and Algebraic Structures; 3) Data Analysis, Statistics, and Probability; 4) Shape, Dimension, and Geometric Relationships. The information in the standards progresses from large to fine grain, detailing specific skills and outcomes students must master: Standards to Prepared Graduate Competencies to Grade Level/Course Expectation to Concepts and Skills Students Master to Evidence Outcomes. The specific indicators of these different levels of mastery are defined in the Evidence Outcomes. It is important not to think of these standards in terms of “introduction, mastery, reinforcement.” All of the evidence outcomes in a certain grade level must be mastered in order for the next higher level of mastery to occur. Again, to maintain consistency and coherence throughout the district, across all levels, adherence to this idea of mastery is vital.

In creating the documents for the 2012 Boulder Valley Curriculum Essentials Documents in mathematics, the writing teams focused on clarity, focus and understanding essential changes from the BVSD 2009 standards to the new 2011 CAS. To maintain the integrity of these documents, it is important that teachers throughout the district follow the standards precisely so that each child in every classroom can be guaranteed a viable education, regardless of the school they attend or if they move from another school, another district or another state. Consistency, clarity and coherence are essential to excellence in mathematics instruction district wide.

Components of the Curriculum Essentials Document

The CED for each grade level and course include the following:

- An At-A-Glance page containing:
  - approximately ten key skills or topics that students will master during the year
  - the general big ideas of the grade/course
  - the Standards of Mathematical Practices
  - assessment tools allow teachers to continuously monitor student progress for planning and pacing needs
  - description of mathematics at that level
- The Grade Level Expectations (GLE) pages. The advanced level courses for high school were based on the high school course with additional topics or more in-depth coverage of topics included in bold text.
- The Grade Level Glossary of Academic Terms lists all of the terms with which *teachers should be familiar and comfortable using during instruction. It is not a comprehensive list of vocabulary for student use.*
- PK-12 Prepared Graduate Competencies
- PK-12 At-A-Glance Guide from the CAS with notes from the CCSS
- CAS Vertical Articulation Guide PK-12

Explanation of Coding

In these documents you will find various abbreviations and coding used by the Colorado Department of Education.
MP – Mathematical Practices Standard
PFL – Personal Financial Literacy
CCSS – Common Core State Standards

> Example: (CCSS: 1.NBT.1) – taken directly from the Common Core State Standards with an reference to the specific CCSS domain, standard and cluster of evidence outcomes.
> NBT – Number Operations in Base Ten
> OA – Operations and Algebraic Thinking
> MD – Measurement and Data
> G – Geometry

Standards for Mathematical Practice from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions,
explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

21^{st} Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21^{st} century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21^{st} century skills, as follows:

Critical Thinking and Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration

Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction

Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

8th Grade Overview

Course Description

M25 is based on the Common Core State Standards and focuses on three critical areas: (1) formulating and reasoning about expressions and equations including solving linear equations and system of equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two-and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Topics at a Glance

- Irrational numbers and radicals
- Positive and negative exponents
- Define functions and compare relations
- Write, graph and use linear equations
- Patterns of association in bivariate data
- Two and Three-dimensional Geometry
- Simultaneous Linear Equations

Assessments

- Formative and summative classroom assessments
- School level assessments
- State level assessment
- District high school transition assessment

Grade Level Expectations

Standard	Big Ideas for Eighth Grade
1. Number Sense, properties, and operations	1. In the real number system, rational and irrational numbers are in one to one correspondence on the number line.
2. Patterns, Functions, & Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically 2. Properties of algebra and equality are used to solve linear equations and systems of equations 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, & Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, & Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity 2. Direct and indirect measurement can be used to describe and make comparisons

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line.

Evidence Outcomes

Students can:

a. Define irrational numbers.[1]
b. Demonstrate informally that every number has a decimal expansion. (CCSS: 8.NS.1)
   i. For rational numbers show that the decimal expansion repeats eventually. (CCSS: 8.NS.1)
   ii. Convert a decimal expansion which repeats eventually into a rational number. (CCSS: 8.NS.1)
c. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.[2] (CCSS: 8.NS.2)
d. Apply the properties of integer exponents to generate equivalent numerical expressions.[3] (CCSS: 8.EE.1)
e. Use square root and cube root symbols to represent solutions to equations of the form *x*2=*p* and *x*3=*p*, where *p* is a positive rational number. (CCSS: 8.EE.2)
f. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.[4] (CCSS: 8.EE.2)
g. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.[5](CCSS: 8.EE.3)
h. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. (CCSS: 8.EE.4)
   i. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.[6] (CCSS: 8.EE.4)
   ii. Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.4)

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. Why are real numbers represented by a number line and why are the integers represented by points on the number line?
2. Why is there no real number closest to zero?
3. What is the difference between rational and irrational numbers?

Relevance and Application:

1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle.
2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density).
3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents.

Nature of Discipline:

1. Mathematics provides a precise language to describe objects and events and the relationships among them.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

[1] Know that numbers that are not rational are called irrational. (CCSS: 8.NS.1)
[2] e.g., *π*2. (CCSS: 8.NS.2)
For example, by truncating the decimal expansion of 2√, show that 2√ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.2)
[3] For example, 32×3–5=3–3=133=127. (CCSS: 8.EE.1)
[4] Know that 2√ is irrational. (CCSS: 8.EE.2)
[5] For example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109, and determine that the world population is more than 20 times larger. (CCSS: 8.EE.3)
[6] e.g., use millimeters per year for seafloor spreading. (CCSS: 8.EE.4)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe the connections between proportional relationships, lines, and linear equations. (CCSS: 8.EE) b. Graph proportional relationships, interpreting the unit rate as the slope of the graph. (CCSS: 8.EE.5) c. Compare two different proportional relationships represented in different ways. [1] (CCSS: 8.EE.5) d. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (CCSS: 8.EE.6) e. Derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.6)	**Inquiry Questions:** 1. How can different representations of linear patterns present different perspectives of situations? 2. How can a relationship be analyzed with tables, graphs, and equations? 3. Why is one variable dependent upon the other in relationships? **Relevance and Application:** 1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates from competing companies or simple interest on savings or credit. 2. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related. 3. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function. **Nature of Discipline:** 1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model. 2. Mathematicians model with mathematics. (MP) [1] For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS: 8.EE.5)

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

2. Properties of algebra and equality are used to solve linear equations and systems of equations.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve linear equations in one variable. (CCSS: 8.EE.7) i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.[2] (CCSS: 8.EE.7a) ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.7b) b. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.8) i. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.8a) ii. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.[3] (CCSS: 8.EE.8b) iii. Solve real-world and mathematical problems leading to two linear equations in two variables.[4] (CCSS: 8.EE.8c)	**Inquiry Questions:** 1. What makes a solution strategy both efficient and effective? 2. How is it determined if multiple solutions to an equation are valid? 3. How does the context of the problem affect the reasonableness of a solution? 4. Why can two equations be added together to get another true equation? **Relevance and Application:** 1. The understanding and use of equations, inequalities, and systems of equations allows for situational analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs. 2. Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs. **Nature of Discipline:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians use appropriate tools strategically. (MP) [2] Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). (CCSS: 8.EE.6a) [3] For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. (CCSS: 8.EE.8b) [4] For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (CCSS: 8.EE.8c)

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Define, evaluate, and compare functions. (CCSS: 8.F) i. Define a function as a rule that assigns to each input exactly one output.[5] (CCSS: 8.F.1) ii. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (CCSS: 8.F.1) iii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).[6] (CCSS: 8.F.2) iv. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line. (CCSS: 8.F.3) v. Give examples of functions that are not linear.[7] b. Use functions to model relationships between quantities. (CCSS: 8.F) i. Construct a function to model a linear relationship between two quantities. (CCSS: 8.F.4) ii. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. (CCSS: 8.F.4) iii. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.4) iv. Describe qualitatively the functional relationship between two quantities by analyzing a graph.[8] (CCSS: 8.F.5) v. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.5) vi. Analyze how credit and debt impact personal financial goals (PFL)	**Inquiry Questions:** 1. How can change best be represented mathematically? 2. Why are patterns and relationships represented in multiple ways? 3. What properties of a function make it a linear function? **Relevance and Application:** 1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation. 2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters. **Nature of Discipline:** 1. Mathematics involves multiple points of view. 2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches. 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP) [5] Function notation is not required in 8th grade. (CCSS: 8.F.11) [6] For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS: 8.F.2) [7] For example, the function *A*=*s*2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. (CCSS: 8.F.3) [8] e.g., where the function is increasing or decreasing, linear or nonlinear. (CCSS: 8.F.5)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics	
Standard: 3. Data Analysis, Statistics, and Probability	
Prepared Graduates: ➢ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.	
GRADE LEVEL EXPECTATION: Eighth Grade	
Concepts and skills students master: 1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge.	
Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. (CCSS: 8.SP.1) b. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.1) c. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. [1] (CCSS: 8.SP.2) d. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. [12](CCSS: 8.SP.3) e. Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. (CCSS: 8.SP.4) i. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. (CCSS: 8.SP.4) ii. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. [3] (CCSS: 8.SP.4)	**Inquiry Questions:** 1. How is it known that two variables are related to each other? 2. How is it known that an apparent trend is just a coincidence? 3. How can correct data lead to incorrect conclusions? 4. How do you know when a credible prediction can be made? **Relevance and Application:** 1. The ability to analyze and interpret data helps to distinguish between false relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation. 2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predicting whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP. **Nature of Discipline:** 1. Mathematicians discover new relationship embedded in information. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) [1] Know that straight lines are widely used to model relationships between two quantitative variables. (CCSS: 8.SP.2) [2] For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (CCSS: 8.SP.3) [3] For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS: 8.SP.4)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

1. Transformations of objects can be used to define the concepts of congruence and similarity.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Verify experimentally the properties of rotations, reflections, and translations.[1] (CCSS: 8.G.1) b. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.3) c. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. (CCSS: 8.G.2) d. Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. (CCSS: 8.G.2) e. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. (CCSS: 8.G.4) f. Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. (CCSS: 8.G.4) g. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.[2] (CCSS: 8.G.5)	**Inquiry Questions:** 1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes? 2. How can you physically verify that two lines are really parallel? **Relevance and Application:** 1. Dilations are used to enlarge or shrink pictures. 2. Rigid motions can be used to make new patterns for clothing or architectural design. **Nature of Discipline:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) [1] Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.1a)Angles are taken to angles of the same measure. (CCSS: 8.G.1b)Parallel lines are taken to parallel lines. (CCSS: 8.G.1c) [2] For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS: 8.G.5

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION: Eighth Grade

Concepts and skills students master:

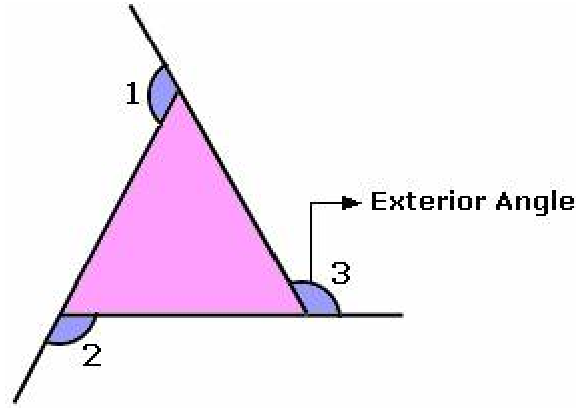
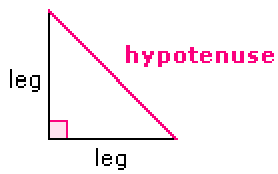
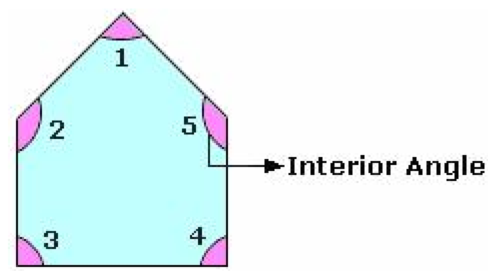
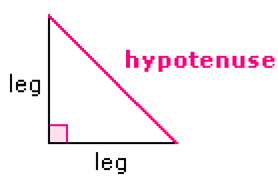
2. Direct and indirect measurement can be used to describe and make comparisons.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.6) b. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.7) c. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.8) d. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.9)	**Inquiry Questions:** 1. Why does the Pythagorean Theorem only apply to right triangles? 2. How can the Pythagorean Theorem be used for indirect measurement? 3. How are the distance formula and the Pythagorean theorem the same? Different? 4. How are the volume formulas for cones, cylinders, prisms and pyramids interrelated? 5. How is volume of an irregular figure measured? 6. How can cubic units be used to measure volume for curved surfaces?
	Relevance and Application: 1. The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings. 2. Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads. 3. The ability to find volume helps to answer important questions such as how to minimize waste by redesigning packaging or maximizing volume by using a circular base.
	Nature of Discipline: 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Geometric objects are abstracted and simplified versions of physical objects 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

Glossary of Terms

Academic Vocabulary
Standard 1: rational numbers, irrational numbers, exponent, power, base numbers, square root, perfect square, cube root, perfect cube, scientific notation
Standard 2: proportional, similar, slope, y-intercept, linear equation, simultaneous equations, function, linear function, non-linear function, rate of change, relation, credit, debt
Standard 3: scatter plots, bivariate, clustering, outliers, association, correlation, linear, non-linear, line of best fit, categorical data, relative frequency, two-way table
Standard 4: congruent, similar, transversal, exterior angle, interior angle, Pythagorean Theorem, hypotenuse, legs, two-dimensional, three-dimensional, cylinder, cone, sphere, pi

Word	**Definition**
Association	A relationship in paired data in which the two sets of data tend to increase together, decrease together or both show no change.
Base number	The number in an exponential expression which is being multiplied by itself. (i.e. For example, 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)
Bivariate	Having two variables.
Categorical data	Variable data fitting into categories (e.g. age group, size).
Clustering	A group of the same or similar elements gathered or occurring closely together.
Cone	A three-dimensional figure with a single, circular base tapering to a single point (vertex).
Congruent	Figures or shapes that are exactly the same size and shape.
Correlation	The degree to which two variables are associated. For example, height and weight have a moderately strong positive correlation.
Credit	An arrangement for deferred payment of a loan or purchase.
Cube root	Cube root is nothing but the one of the identical factors of a given number. A number that must be multiplied times itself three times to equal the number inside the radicand. For example, $\sqrt[3]{8} = 2$; 2 is the cube root of 8, since $2^3 = 8$.
Cylinder	A three-dimensional figure with two congruent, parallel, circular bases.
Debt	An amount owed. There is usually an associated interest.
Exponent	A number used to tell how many times a number or variable is used as a factor. (i.e., 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)

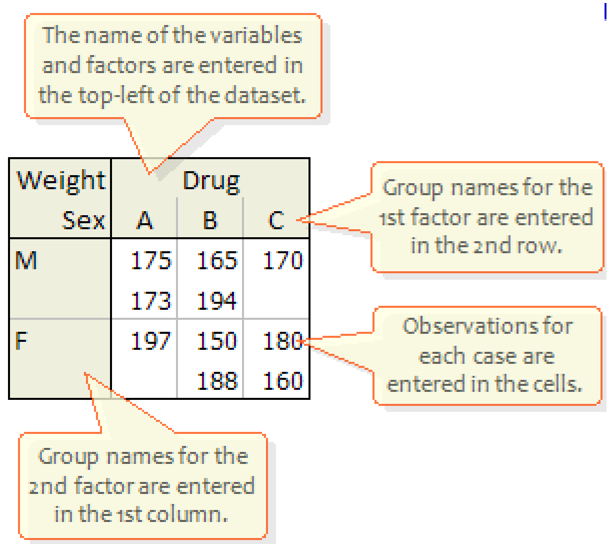
Exterior angle	An angle formed by one side of a polygon and the extension of an adjacent side. 
Function	Function is a relation in which each x value (element of the domain) is paired with exactly one y value (element of the range).
Hypotenuse	The side of a right triangle opposite the right angle. 
Interior angle	An angle in the interior of a plane figure. 
Irrational numbers	The set of numbers which cannot be represented as fractions. (i.e $\sqrt[3]{29}$, ∏, e, and $\sqrt{7}$).
Legs	The sides of a right triangle opposite an acute angle or the two sides in a right triangle that form the right angle. 
Linear equation	Linear equation is an equation of the form $Ax + By = C$, where $A \neq 0$ and $B \neq 0$. The graph of a linear equation is a straight line.
Line-of-best-fit	A line, segment or ray drawn on a scatter plot to estimate the relationship between two sets of data.
Linear	A graph or set of data that can be modeled by a line.
Linear function	A function that can be graphically represented in the Cartesian coordinate plane by a straight line is called a Linear Function. A linear function is a first degree polynomial of the form, $F(x) = m\ x + c$, where m and care constants and x is a real variable.
Nonlinear	A graph or set of data that cannot be modeled by a straight line.
Nonlinear function	A function with a corresponding graph which is a curve.
Outliers	A data point that is distinctly separate from the rest of the data.
Perfect cube	Any number that is the cube of a rational number. For example, 64 is a perfect cube, since $4^3 = 64$

Perfect square	Any number that is the square of a rational number. For example, 49 is a perfect square, since $7^2 = 49$
Pi	The ratio of the circumference of any circle to its diameter. $\pi \approx 3.14$
Power	See Exponent. A number used to tell how many times a number or variable is used as a factor. (i.e., 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)
Proportional	Two sets of numbers are proportional if one set is a constant times the other. A proportion is an equation written in the form $\frac{a}{b} = \frac{c}{d}$ stating that two ratios are equivalent.
Pythagorean Theorem	An equation relating the lengths of the sides of a right triangle. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.
Rate of change	The change in one variable compared to the change in another. Frequently written as a fraction/proportion.
Rational numbers	A number that can be expressed in the form a/b, where a and b are integers and b ≠ 0, for example, 3/4, 2/1, or 11/3. Every integer is a rational number. Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.
Relation	A set of ordered pairs.
Relative frequency	Relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful events.
Scatterplot	A graph of paired data in which the data values are plotted as (x,y) points.
Similar/similarity	Informally: Figures with the exact same shape but maybe different sizes. Formally: Figures whose corresponding sides are proportional and whose corresponding angles are congruent. If two figures are similar, we say that there is similarity between the figures.
Simultaneous equations	Two or more equations containing common variables. For any linear system, exactly one of the following will be true: There is only one solution, infinitely many solutions, or there are no solutions.
Slope	Slope is the measure of steepness of a line. Slope $= \frac{\text{the change in the } y\text{-coordinates}}{\text{the change in the } x\text{-coordinates}} = \frac{rise}{run}$.
Sphere	A three-dimensional figure consisting of all points equidistant from a single point.
Square root	The number that when multiplied by itself results in a given number (e.g., The square root of 36 is 6 because 6 x 6 = 36).
Three-dimensional	The property of space having length, width and height.
Transversal	A line that intersects two or more lines or sides of a plane figure.
Two-dimensional	The property of a plane figure having length in two perpendicular directions (e.g. a square, triangle).
Two-way table	A table consisting of rows and columns that represent two categorical variables. Also called a contingency table. Useful tool for examining relationships between categorical variables. Also called a situation matrix, often used to determine the total number of combinations. **Example Question** Emma has collected information about the cats and dogs that children in her class have as pets. There are 30 pupils. For each pupil, there are four possible responses they could make: • The pupil has a cat and a dog. • The pupil has a cat but not a dog. • The pupil has a dog but not a cat.

- The pupil does not have a cat or a dog.

This information can be represented in the table below.

	Has a dog	Does not have a dog
Has a cat	*These pupils have both a cat and a dog*	*These pupils have a cat but not a dog*
Does not have a cat	*These pupils have a dog but not a cat*	*These pupils do not have a cat or a dog*



y-intercept	A point at which a graph intersects the y-axis.

Definitions adapted from:

Boulder Valley School District Curriculum Essentials Document, 2009.

“Math Dictionary” www.icoachmath.com/math_dictionary/mathdictionarymain.html. Copyright © 1999 - 2011 HighPoints Learning Inc. December 30, 2011.

“Arranging a Two-Way Table Data Set” http://analyse-it.biz/support/documentation/220/using_analyse-it/preparing_data_in_excel/arranging_a_2-way_table_dataset.htm Analyse-it® and Analyze-it® are trademarks of Analyse-it Software, Copyright © 1997-2011. December 30, 2011.

“Unit 1 Section 2: Two Way Tables” http://www.cimt.plymouth.ac.uk/projects/mepres/book7/bk7i1/bk7_1i2.html. Produced by A.J.Reynolds January 2001. December 30, 2011.

PK-12 Alignment of Mathematical Standards

The following pages will provide teachers with an understanding of the alignment of the standards from Pre-Kindergarten through High School. An understanding of this alignment and each grade level's role in assuring that each student graduates with a thorough understanding of the standards at each level is an important component of preparing our students for success in the 21st century. Provided in this section are the Prepared Graduate Competencies in Mathematics, an At-a-glance description of the Grade Level Expectations for each standard at each grade level, and a thorough explanation from the CCSS about the alignment of the standards across grade levels.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Mathematics
Prepared Graduate Competencies at Grade Levels
PK-12 Scope and Sequence

Understand the structure and properties of our number system. At the most basic level numbers are abstract symbols that represent real-world quantities.		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.1-GLE.1	The complex number system includes real numbers and imaginary numbers
Eighth Grade	MA10-GR.8-S.1-GLE.1	In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
Sixth Grade	MA10-GR.6-S.1-GLE.3	In the real number system, rational numbers have a unique location on the number line and in space
Fifth Grade	MA10-GR.5-S.1-GLE.1	The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms
	MA10-GR.5-S.1-GLE.4	The concepts of multiplication and division can be applied to multiply and divide fractions
Fourth Grade	MA10-GR.4-S.1-GLE.1	The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms
Third Grade	MA10-GR.3-S.1-GLE.1	The whole number system describes place value relationships and forms the foundation for efficient algorithms
Second Grade	MA10-GR.2-S.1-GLE.1	The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms
First Grade	MA10-GR.1-S.1-GLE.1	The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms
Kindergarten	MA10-GR.K-S.1-GLE.1	Whole numbers can be used to name, count, represent, and order quantity

Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error.		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.1-GLE.2	Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
Seventh Grade	MA10-GR.7-S.4-GLE.2	Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure
Fifth Grade	MA10-GR.5-S.4-GLE.1	Properties of multiplication and addition provide the foundation for volume an attribute of solids.
Fourth Grade	MA10-GR.4-S.4-GLE.1	Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time
Third Grade	MA10-GR.3-S.4-GLE.2	Linear and area measurement are fundamentally different and require different units of measure
	MA10-GR.3-S.4-GLE.3	Time and attributes of objects can be measured with appropriate tools
Second Grade	MA10-GR.2-S.4-GLE.2	Some attributes of objects are measurable and can be quantified using different tools
First Grade	MA10-GR.1-S.4-GLE.2	Measurement is used to compare and order objects and events
Kindergarten	MA10-GR.K-S.4-GLE.2	Measurement is used to compare and order objects
Preschool	MA10-GR.P-S.1-GLE.1	Quantities can be represented and counted
	MA10-GR.P-S.4-GLE.2	Measurement is used to compare objects

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.4	Solutions to equations, inequalities and systems of equations are found using a variety of tools
Eight Grade	MA10-GR.8-S.2-GLE.2	Properties of algebra and equality are used to solve linear equations and systems of equations
Seventh Grade	MA10-GR.7-S.1-GLE.2	Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
Sixth Grade	MA10-GR.6-S.1-GLE.2	Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency
Fifth Grade	MA10-GR.5-S.1-GLE.2	Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency
	MA10-GR.5-S.1-GLE.3	Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency
Fourth Grade	MA10-GR.4-S.1-GLE.3	Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency
Third Grade	MA10-GR.3-S.1-GLE.3	Multiplication and division are inverse operations and can be modeled in a variety of ways
Second Grade	MA10-GR.2-S.1-GLE.2	Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.		
Grade Level	**Numbering System**	**Grade Level Expectations**
Seventh Grade	MA10-GR.7-S.1-GLE.1	Proportional reasoning involves comparisons and multiplicative relationships among ratios
Sixth Grade	MA10-GR.6-S.1-GLE.1	Quantities can be expressed and compared using ratios and rates

Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.3-GLE.3	Probability models outcomes for situations in which there is inherent randomness
Seventh Grade	MA10-GR.7-S.3-GLE.2	Mathematical models are used to determine probability

Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.3-GLE.1	Visual displays and summary statistics condense the information in data sets into usable knowledge
Eighth Grade	MA10-GR.8-S.3-GLE.1	Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
Sixth Grade	MA10-GR.6-S.3-GLE.1	Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge
Fifth Grade	MA10-GR.5-S.3-GLE.1	Visual displays are used to interpret data
Fourth Grade	MA10-GR.4-S.3-GLE.1	Visual displays are used to represent data
Third Grade	MA10-GR.3-S.3-GLE.1	Visual displays are used to describe data
Second Grade	MA10-GR.2-S.3-GLE.1	Visual displays of data can be constructed in a variety of formats to solve problems
First Grade	MA10-GR.1-S.3-GLE.1	Visual displays of information can used to answer questions

Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.3	Expressions can be represented in multiple, equivalent forms
High School	MA10-GR.HS-S.2-GLE.1	Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically
Seventh Grade	MA10-GR.7-S.2-GLE.1	Properties of arithmetic can be used to generate equivalent expressions
Fourth Grade	MA10-GR.4-S.1-GLE.2	Different models and representations can be used to compare fractional parts
Third Grade	MA10-GR.3-S.1-GLE.2	Parts of a whole can be modeled and represented in different ways

Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.1	Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables
Fifth Grade	MA10-GR.5-S.2-GLE.1	Number patterns are based on operations and relationships
Fourth Grade	MA10-GR.4-S.2-GLE.1	Number patterns and relationships can be represented by symbols
Preschool	MA10-GR.P-S.4-GLE.1	Shapes can be observed in the world and described in relation to one another

Apply transformation to numbers, shapes, functional representations, and data		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.4-GLE.1	Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically
High School	MA10-GR.HS-S.4-GLE.3	Objects in the plane can be described and analyzed algebraically
Eighth Grade	MA10-GR.8-S.4-GLE.1	Transformations of objects can be used to define the concepts of congruence and similarity
Seventh Grade	MA10-GR.7-S.4-GLE.1	Modeling geometric figures and relationships leads to informal spatial reasoning and proof
Second Grade	MA10-GR.2-S.4-GLE.1	Shapes can be described by their attributes and used to represent part/whole relationships
First Grade	MA10-GR.1-S.1-GLE.2	Number relationships can be used to solve addition and subtraction problems
Kindergarten	MA10-GR.K-S.1-GLE.2	Composing and decomposing quantity forms the foundation for addition and subtraction

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.4-GLE.4	Attributes of two- and three-dimensional objects are measurable and can be quantified
Sixth Grade	MA10-GR.6-S.2-GLE.1	Algebraic expressions can be used to generalize properties of arithmetic
	MA10-GR.6-S.2-GLE.2	Variables are used to represent unknown quantities within equations and inequalities
	MA10-GR.6-S.4-GLE.1	Objects in space and their parts and attributes can be measured and analyzed
Fifth Grade	MA10-GR.5-S.4-GLE.2	Geometric figures can be described by their attributes and specific locations in the plane
Fourth Grade	MA10-GR.4-S.4-GLE.2	Geometric figures in the plane and in space are described and analyzed by their attributes
Third Grade	MA10-GR.3-S.4-GLE.1	Geometric figures are described by their attributes
First Grade	MA10-GR.1-S.4-GLE.1	Shapes can be described by defining attributes and created by composing and decomposing
Kindergarten	MA10-GR.K-S.4-GLE.1	Shapes can be described by characteristics and position and created by composing and decomposing

Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
This prepared graduate competency is addressed through all of the grade level expectations and is part of the mathematical practices.

Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.2	Quantitative relationships in the real world can be modeled and solved using functions
	MA10-GR.HS-S.3-GLE.2	Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions
	MA10-GR.HS-S.4-GLE.2	Concepts of similarity are foundational to geometry and its applications
	MA10-GR.HS-S.4-GLE.5	Objects in the real world can be modeled using geometric concepts
Eighth Grade	MA10-GR.8-S.2-GLE.3	Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
	MA10-GR.8-S.4-GLE.2	Direct and indirect measurement can be used to describe and make comparisons
Seventh Grade	MA10-GR.7-S.2-GLE.2	Equations and expressions model quantitative relationships and phenomena
	MA10-GR.7-S.3-GLE.1	Statistics can be used to gain information about populations by examining samples

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
High School	
1. Number Sense, Properties, and Operations	1. The complex number system includes real numbers and imaginary numbers 2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
2. Patterns, Functions, and Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 2. Quantitative relationships in the real world can be modeled and solved using functions 3. Expressions can be represented in multiple, equivalent forms 4. Solutions to equations, inequalities and systems of equations are found using a variety of tools
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics condense the information in data sets into usable knowledge 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions 3. Probability models outcomes for situations in which there is inherent randomness
4. Shape, Dimension, and Geometric Relationships	1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically 2. Concepts of similarity are foundational to geometry and its applications 3. Objects in the plane can be described and analyzed algebraically 4. Attributes of two- and three-dimensional objects are measurable and can be quantified 5. Objects in the real world can be modeled using geometric concepts

From the Common State Standards for Mathematics, Pages 58, 62, 67, 72-74, and 79.

Mathematics | High School—Number and Quantity
Numbers and Number Systems. *During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.*

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. *In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.*

Mathematics | High School—Algebra

Expressions. *An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.*

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. *An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.*

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. *Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.*

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the

intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- *Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.*
- *Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.*
- *Designing the layout of the stalls in a school fair so as to raise as much money as possible.*
- *Analyzing stopping distance for a car.*
- *Modeling savings account balance, bacterial colony growth, or investment growth.*
- *Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.*
- *Analyzing risk in situations such as extreme sports, pandemics, and terrorism.*
- *Relating population statistics to individual predictions.*

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

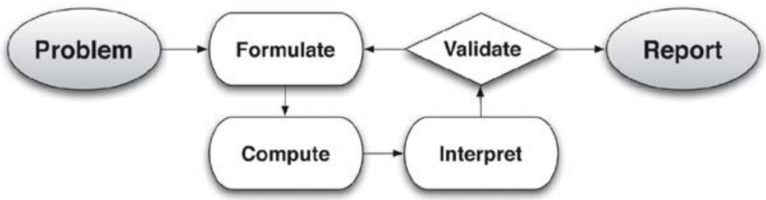
The basic modeling cycle is summarized in the diagram (below). It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards*. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*



Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. *The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.*

Mathematics | High School—Statistics and Probability*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume *various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.*

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

***Connections to Functions and Modeling**. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.*

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Eighth Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically 2. Properties of algebra and equality are used to solve linear equations and systems of equations 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity 2. Direct and indirect measurement can be used to describe and make comparisons

From the Common State Standards for Mathematics, Page 52.

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can

translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Seventh Grade	
1. Number Sense, Properties, and Operations	1. Proportional reasoning involves comparisons and multiplicative relationships among ratios 2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
2. Patterns, Functions, and Algebraic Structures	1. Properties of arithmetic can be used to generate equivalent expressions 2. Equations and expressions model quantitative relationships and phenomena
3. Data Analysis, Statistics, and Probability	1. Statistics can be used to gain information about populations by examining samples 2. Mathematical models are used to determine probability
4. Shape, Dimension, and Geometric Relationships	1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof 2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

From the Common State Standards for Mathematics, Page 46.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve

a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Sixth Grade	
1. Number Sense, Properties, and Operations	1. Quantities can be expressed and compared using ratios and rates 2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency 3. In the real number system, rational numbers have a unique location on the number line and in space
2. Patterns, Functions, and Algebraic Structures	1. Algebraic expressions can be used to generalize properties of arithmetic 2. Variables are used to represent unknown quantities within equations and inequalities
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Objects in space and their parts and attributes can be measured and analyzed

From the Common State Standards for Mathematics, Pages 39-40

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that

the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios,
and they use equations (such as 3x = y) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Fifth Grade	
1. Number Sense, Properties, and Operations	1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms 2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency 3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency 4. The concepts of multiplication and division can be applied to multiply and divide fractions
2. Patterns, Functions, and Algebraic Structures	1. Number patterns are based on operations and relationships
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to interpret data
4. Shape, Dimension, and Geometric Relationships	1. Properties of multiplication and addition provide the foundation for volume an attribute of solids 2. Geometric figures can be described by their attributes and specific locations in the plane

From the Common State Standards for Mathematics, Page 33.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Fourth Grade	
1. Number Sense, Properties, and Operations	1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms 2. Different models and representations can be used to compare fractional parts 3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency
2. Patterns, Functions, and Algebraic Structures	1. Number patterns and relationships can be represented by symbols
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to represent data
4. Shape, Dimension, and Geometric Relationships	1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time 2. Geometric figures in the plane and in space are described and analyzed by their attributes

From the Common State Standards for Mathematics, Page 27.

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Third Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships and forms the foundation for efficient algorithms 2. Parts of a whole can be modeled and represented in different ways 3. Multiplication and division are inverse operations and can be modeled in a variety of ways
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to describe data
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures are described by their attributes 2. Linear and area measurement are fundamentally different and require different units of measure 3. Time and attributes of objects can be measured with appropriate tools

From the Common State Standards for Mathematics, Page 21.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Second Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms 2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays of data can be constructed in a variety of formats to solve problems
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be described by their attributes and used to represent part/whole relationships 2. Some attributes of objects are measurable and can be quantified using different tools

From the Common State Standards for Mathematics, Page 17.

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
First Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms 2. Number relationships can be used to solve addition and subtraction problems
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays of information can be used to answer questions
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be described by defining attributes and created by composing and decomposing 2. Measurement is used to compare and order objects and events

From the Common State Standards for Mathematics, Page 13.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry

[1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Kindergarten	
1. Number Sense, Properties, and Operations	1. Whole numbers can be used to name, count, represent, and order quantity 2. Composing and decomposing quantity forms the foundation for addition and subtraction
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes are described by their characteristics and position and created by composing and decomposing 2. Measurement is used to compare and order objects

From the Common State Standards for Mathematics, Page 9.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Preschool	
1. Number Sense, Properties, and Operations	1. Quantities can be represented and counted
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be observed in the world and described in relation to one another 2. Measurement is used to compare objects