Boulder Valley School District
Excellence and Equity

M30 Pre-Algebra Curriculum Essentials Document

Boulder Valley School District
Department of Curriculum and Instruction
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Boulder Valley School District Mathematics – An Introduction to The Curriculum Essentials Document

Background

The 2009 Common Core State Standards (CCSS) have brought about a much needed move towards consistency in mathematics throughout the state and nation. In December 2010, the Colorado Academic Standards revisions for Mathematics were adopted by the State Board of Education. These standards aligned the previous state standards to the Common Core State Standards to form the Colorado Academic Standards (CAS). The CAS include additions or changes to the CCSS needed to meet state legislative requirements around Personal Financial Literacy.

The Colorado Academic Standards Grade Level Expectations (GLE) for math are being adopted in their entirety and without change in the PK-8 curriculum. This decision was made based on the thorough adherence by the state to the CCSS. These new standards are specific, robust and comprehensive. Additionally, the essential linkage between the standards and the proposed 2014 state assessment system, which may include interim, formative and summative assessments, is based specifically on these standards. The overwhelming opinion amongst the mathematics teachers, school and district level administration and district level mathematics coaches clearly indicated a desire to move to the CAS without creating a BVSD version through additions or changes.

The High School standards provided to us by the state did not delineate how courses should be created. Based on information regarding the upcoming assessment system, the expertise of our teachers and the writers of the CCSS, the decision was made to follow the recommendations in the ***Common Core State Standards for Mathematics- Appendix A: Designing High School Math Courses Based on the Common Core State Standards***. The writing teams took the High School CAS and carefully and thoughtfully divided them into courses for the creation of the 2012 BVSD Curriculum Essentials Documents (CED).

The Critical Foundations of the 2011 Standards

The expectations in these documents are based on mastery of the topics at specific grade levels with the understanding that the standards, themes and big ideas reoccur throughout PK-12 at varying degrees of difficulty, requiring different levels of mastery. The Standards are: 1) Number Sense, Properties, and Operations; 2) Patterns, Functions, and Algebraic Structures; 3) Data Analysis, Statistics, and Probability; 4) Shape, Dimension, and Geometric Relationships. The information in the standards progresses from large to fine grain, detailing specific skills and outcomes students must master: Standards to Prepared Graduate Competencies to Grade Level/Course Expectation to Concepts and Skills Students Master to Evidence Outcomes. The specific indicators of these different levels of mastery are defined in the Evidence Outcomes. It is important not to think of these standards in terms of "introduction, mastery, reinforcement." All of the evidence outcomes in a certain grade level must be mastered in order for the next higher level of mastery to occur. Again, to maintain consistency and coherence throughout the district, across all levels, adherence to this idea of mastery is vital.

In creating the documents for the 2012 Boulder Valley Curriculum Essentials Documents in mathematics, the writing teams focused on clarity, focus and understanding essential changes from the BVSD 2009 standards to the new 2011 CAS. To maintain the integrity of these documents, it is important that teachers throughout the district follow the standards precisely so that each child in every classroom can be guaranteed a viable education, regardless of the school they attend or if they move from another school, another district or another state. Consistency, clarity and coherence are essential to excellence in mathematics instruction district wide.

Components of the Curriculum Essentials Document

The CED for each grade level and course include the following:

- An At-A-Glance page containing:
  - approximately ten key skills or topics that students will master during the year
  - the general big ideas of the grade/course
  - the Standards of Mathematical Practices
  - assessment tools allow teachers to continuously monitor student progress for planning and pacing needs
  - description of mathematics at that level
- The Grade Level Expectations (GLE) pages. The advanced level courses for high school were based on the high school course with additional topics or more in-depth coverage of topics included in bold text.
- The Grade Level Glossary of Academic Terms lists all of the terms with which *teachers should be familiar and comfortable using during instruction. It is not a comprehensive list of vocabulary for student use.*
- PK-12 Prepared Graduate Competencies
- PK-12 At-A-Glance Guide from the CAS with notes from the CCSS
- CAS Vertical Articulation Guide PK-12

Explanation of Coding

In these documents you will find various abbreviations and coding used by the Colorado Department of Education.

MP – Mathematical Practices Standard
PFL – Personal Financial Literacy
CCSS – Common Core State Standards

> Example: (CCSS: 1.NBT.1) – taken directly from the Common Core State Standards with an reference to the specific CCSS domain, standard and cluster of evidence outcomes.
> NBT – Number Operations in Base Ten
> OA – Operations and Algebraic Thinking
> MD – Measurement and Data
> G – Geometry

Standards for Mathematical Practice from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences *between different approaches.*

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

<u>Critical Thinking and Reasoning</u>
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

<u>Information Literacy</u>
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

<u>Collaboration</u>
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

<u>Self-Direction</u>
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

<u>Invention</u>
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

M30 Pre Algebra Overview

Course Description

This is an accelerated course that teaches the middle level topics covered in M05 and M15 in preparing students for M25 and Algebra 1. Students will cover number operations, introductory probability and statistics, coordinate geometry and rate of change through linear algebra.

Topics at a Glance

- Ratios and Proportional Reasoning
- Operations with Rational Numbers and Integers
- Coordinate Geometry
- Summary Statistics and Displays
- Algebraic and Equivalent Expressions
- Modeling with Equations and Expressions
- Samples and Populations
- Modeling Geometric
- Area, Volume, Surface Area
- Introduction to linear algebra

Assessments

- Middle School State Level assessments
- District placement assessment
- Teacher created assessments

Grade Level Expectations

Standard	Big Ideas for Seventh Grade
1. Number Sense, properties, and operations	1. The complex number system includes real numbers and imaginary numbers 2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
2. Patterns, Functions, & Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 2. Quantitative relationships in the real world can be modeled and solved using functions 3. Expressions can be represented in multiple, equivalent forms 4. Solutions to equations, inequalities and systems of equations are found using a variety of tools
3. Data Analysis, Statistics, & Probability	1. 1 Visual displays and summary statistics condense the information in data sets into usable knowledge 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions 3. Probability models outcomes for situations in which there is inherent randomness
4. Shape, Dimension, & Geometric Relationships	1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically 2. Concepts of similarity are foundational to geometry and its applications 3. Objects in the plane can be described and analyzed algebraically 4. Attributes of two- and three-dimensional objects are measurable and can be quantified 5. Objects in the real world can be modeled using geometric concepts

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision. 7.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*** BOLD* Indicates 7th grade CAS standards**

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.

GRADE LEVEL / COURSE EXPECTATION: Pre Algebra

Concepts and skills students master:

1. Quantities can be expressed and compared using ratios and rates.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.[1] (CCSS: 6.RP.1) b. Apply the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.[2] (CCSS: 6.RP.2) c. Use ratio and rate reasoning to solve real-world and mathematical problems.[3] (CCSS: 6.RP.3) i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. (CCSS: 6.RP.3a) ii. Use tables to compare ratios. (CCSS: 6.RP.3a) iii. Solve unit rate problems including those involving unit pricing and constant speed.[4] (CCSS: 6.RP.3b) iv. Find a percent of a quantity as a rate per 100.[5] (CCSS: 6.RP.3c) v. Solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.3c) vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions.* (PFL) vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving. (PFL) viii. Use ratio reasoning to convert measurement units.[6] (CCSS:6.RP.3d)	**Inquiry Questions:** 1. How are ratios different from fractions? 2. What is the difference between quantity and number? **Relevance and Application:** 1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions. 2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers. 3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks. **Nature of Discipline:** 1. Mathematicians develop simple procedures to express complex mathematical concepts. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP) [1] For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” (CCSS: 6.RP.1) [2] For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” (CCSS: 6.RP.2) [3] e.g., By reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.3) [4] For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (CCSS: 6.RP.3b) [5] e.g., 30% of a quantity means 30/100 times the quantity. (CCSS: 6.RP.3c) * Using simple interest to compare rates in a basic comparison. For example, if bank A offers

	6.5% interest and bank B offers 6.75%, which bank offers you a better rate for your $1,000 investment? [6] Manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.3d)

Content Area: Mathematics	
Standard: 1. Number Sense, Properties, and Operations	
Prepared Graduates: ➢ Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.	
GRADE LEVEL / COURSE EXPECTATION: Pre Algebra	
Concepts and skills students master: 2. Proportional reasoning involves comparisons and multiplicative relationships among ratios.	
Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: **a. Analyze proportional relationships and use them to solve real-world and mathematical problems.(CCSS: 7.RP)** **b. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.[1] (CCSS: 7.RP.1)** **c. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.2)** **i. Determine whether two quantities are in a proportional relationship.[2] (CCSS: 7.RP.2a)** **ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS:7.RP.2b)** **iii. Represent proportional relationships by equations.[3]** **ii. (CCSS: 7.RP.2c)** **i. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (CCSS: 7.RP.2d)** **d. Use proportional relationships to solve multistep ratio and percent problems.[4] (CCSS: 7.RP.3)** **i. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality. (PFL)** ii. **Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease. (PFL)**	**Inquiry Questions:** 1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison? 2. What comparisons can be made using ratios? 3. How do you know when a proportional relationship exists? 4. How can proportion be used to argue fairness? 5. When is it better to use an absolute comparison? 6. When is it better to use a relative comparison? **Relevance and Application:** 1. The use of ratios, rates, and proportions allows sound decision making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies. 2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories. 3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature Of Discipline:** 1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) [1] For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 12/14 miles per hour, equivalently 2 miles per hour. (CCSS:

	7.RP.1) [2] e.g., By testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.2a) [3] For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. (CCSS: 7.RP.2c) [4] Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS: 7.RP.3)

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL / COURSE EXPECTATION: Pre Algebra

Concepts and skills students master:

3. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. (CCSS: 6.NS.3) b. Find the greatest common factor of two whole numbers less than or equal to 100. (CCSS: 6.NS.4) c. Find the least common multiple of two whole numbers less than or equal to 12. (CCSS: 6.NS.4) d. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.[7] (CCSS: 6.NS.4) e. Interpret and model quotients of fractions through the creation of story contexts.[8] (CCSS: 6.NS.1) f. Compute quotients of fractions.[9] (CCSS: 6.NS.1) g. Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.[10] (CCSS: 6.NS.1) **h. Apply understandings of addition and subtraction to add and subtract rational numbers including integers. (CCSS: 7.NS.1)** **i. Represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.1)** **ii. Describe situations in which opposite quantities combine to make 0.[5] (CCSS: 7.NS.1a)** **iii. Demonstrate p + q as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. (CCSS: 7.NS.1b)** **iv. Show that a number and its opposite have a sum of 0 (are additive inverses). (CCSS: 7.NS.1b)**	**Inquiry Questions:** 1. Why might estimation be better than an exact answer? 2. How do operations with fractions and decimals compare to operations with whole numbers? **3. How do operations with rational numbers compare to operations with integers?** **4. How do you know if a computational strategy is sensible?** **5. Is 0.9 repeating equal to one?** **6. How do you know whether a fraction can be represented as a repeating or terminating decimal?** **Relevance and Application:** 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry. 2. **The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax.** 3. **Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns.** 4. **People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages).** **Nature of Discipline:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians model with mathematics. (MP) **3. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to**

v. Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.1c) **vi. Demonstrate subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). (CCSS: 7.NS.1c)** **vii. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.1c)** **viii. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.1d)** i. **Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers including integers. (CCSS: 7.NS.2)** **i. Apply properties of operations to multiplication of rational numbers.[6] (CCSS: 7.NS.2a)** **ii. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.2a)** **iii. Apply properties of operations to divide integers.[7] (CCSS: 7.NS.2b)** **iv. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.2c)** **v. Convert a rational number to a decimal using long division. (CCSS: 7.NS.2d)** **vi. Show that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.2d)** j. **Solve real-world and mathematical problems involving the four operations with rational numbers.[8] (CCSS: 7.NS.3)**	**accomplish various tasks.** **4. Mathematicians make sense of problems and persevere in solving them. (MP)** **5. Mathematicians construct viable arguments and critique the reasoning of others. (MP)** **6. Mathematicians look for and make use of structure. (MP)** [7] For example, express 36 + 8 as 4 (9 + 2). (CCSS: 6.NS.4) [8] For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (CCSS: 6.NS.1) [9] In general, (a/b) ÷ (c/d) = ad/bc.). (CCSS: 6.NS.1) [10] How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? (CCSS: 6.NS.1) **[5] For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. (CCSS: 7.NS.1a)** **[6] Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. (CCSS: 7.NS.2a)** **[7] Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). (CCSS: 7.NS.2b) Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.2b)** **[8] Computations with rational numbers extend the rules for manipulating fractions to complex fractions. (CCSS: 7.NS.3)** *** BOLD* Indicates 7th grade CAS standards**

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.

GRADE LEVEL / COURSE EXPECTATION: Pre Algebra

Concepts and skills students master:

4. In the real number system, rational numbers have a unique location on the number line and in space.

Evidence Outcomes

Students can:

a. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values.[11] (CCSS: 6.NS.5)
   i. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.5)

b. Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.[12] (CCSS: 6.NS.6)
   i. Describe a rational number as a point on the number line. (CCSS: 6.NS.6)
   ii. Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line. (CCSS: 6.NS.6a)
   iii. Identify that the opposite of the opposite of a number is the number itself.[13] (CCSS: 6.NS.6a)
   iv. Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.6b)
   v. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. (CCSS: 6.NS.6c)
   vi. Find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.6c)

c. Order and find absolute value of rational numbers. (CCSS: 6.NS.7)
   i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.[14] (CCSS: 6.NS.7a)
   ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts. [15] (CCSS: 6.NS.7b)
   iii. Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.[16] (CCSS: 6.NS.7c)

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. Why are there negative numbers?
2. How do we compare and contrast numbers?
3. Are there more rational numbers than integers?

Relevance and Application:

1. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends.
2. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down

Nature of Discipline:

1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians attend to precision. (MP)

[11] e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). (CCSS: 6.NS.5)
[12] Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. (CCSS: 6.NS.6)
[13] e.g., –(–3) = 3, and that 0 is its own opposite. (CCSS: 6.NS.6a)
[14] For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. (CCSS: 6.NS.7a)
[15] For example, write –3*o*C > –7*o* C to express the fact that –3*o* C is warmer than –7*o* C. (CCSS: 6.NS.7b)
[16] For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. (CCSS: 6.NS.7c)
[17] For example, recognize that an account balance less than –30 dollars represents a

iv. Distinguish comparisons of absolute value from statements about order.[17] (CCSS: 6.NS.7d) d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.8)	debt greater than 30 dollars. (CCSS: 6.NS.7d)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 2: Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

1. Algebraic expressions can be used to generalize properties of arithmetic.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students Can: a. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.1) b. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.2) i. Write expressions that record operations with numbers and with letters standing for numbers.[1] (CCSS: 6.EE.2a) ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity.[2] (CCSS: 6.EE.2b) iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems.[3] (CCSS: 6.EE.2c) iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (CCSS: 6.EE.2c) c. Apply the properties of operations to generate equivalent expressions.[4] (CCSS: 6.EE.3) d. Identify when two expressions are equivalent.[5] (CCSS: 6.EE.4)	**Inquiry Questions:** 1. If we didn't have variables, what would we use? 2. What purposes do variable expressions serve? 3. What are some advantages to being able to describe a pattern using variables? 4. Why does the order of operations exist? 5. What other tasks/processes require the use of a strict order of steps? **Relevance and Application:** 1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts. 2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies. **Nature of Discipline:** 1. Mathematicians use place value to represent many numbers with only ten digits. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP) [1] For example, express the calculation "Subtract y from 5" as 5 – y. (CCSS: 6.EE.2a) [2] For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. (CCSS: 6.EE.2b) [3] For example, use the formulas $V=s^3$ and $A=6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2. (CCSS: 6.EE.2c) [4] For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. (CCSS: 6.EE.3)

	[5] i.e., When the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities. (CCSS: 6.EE.4)

Content Area: Mathematics

Standard: 2: Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

2. Properties of arithmetic can be used to generate equivalent expressions.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use properties of operations to generate equivalent expressions. (CCSS: 7.EE) i. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.1) ii. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.[1] (CCSS: 7.EE.2)	**Inquiry Questions:** 1. How do symbolic transformations affect an equation or expression? 2. How is it determined that two algebraic expressions are equivalent? **Relevance and Application:** 1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass. **Nature of Discipline:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP) [1]For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." (CCSS: 7.EE.2)

Content Area: Mathematics

Standard: 2: Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

3. Variables are used to represent unknown quantities within equations and inequalities.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (CCSS: 6.EE.5) b. Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.5) c. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. (CCSS: 6.EE.6) i. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.6) d. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.* (CCSS: 6.EE.7) e. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. (CCSS: 6.EE.8) f. Show that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.8) g. Represent and analyze quantitative relationships between dependent and independent variables. (CCSS: 6.EE) i. Use variables to represent two quantities in a real-world problem that change in relationship to one another. (CCSS: 6.EE.9) ii. Write an equation to express one quantity, thought	**Inquiry Questions:** 1. Do all equations have exactly one unique solution? Why? 2. How can you determine if a variable is independent or dependent? **Relevance & Application:** 1. Variables allow communication of big ideas with very few symbols. For example, d=r•t is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and C=π•d expresses the relationship between circumference and diameter of a circle. 2. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change. **Nature Of Discipline:** 1. Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables. 2. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable. 3. Mathematicians model with mathematics. (MP) [6]For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time. (CCSS: 6.EE.9) ** Negative values for p and q may be used if students have easily gained mastery of the concepts of integers and the solving equations via additive and multiplicative inverses.*

of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (CCSS: 6.EE.9) iii. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.[6] (CCSS: 6.EE.9)	

Content Area: Mathematics	
Standard: 2: Patterns, Functions, and Algebraic Structures	
Prepared Graduates: ➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.	
GRADE LEVEL EXPECTATION: Pre Algebra	
Concepts and skills students master: 4. Equations and expressions model quantitative relationships and phenomena.	
Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form,[2] using tools strategically. (CCSS: 7.EE.3) b. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.[3] (CCSS: 7.EE.3) c. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.4) i. Fluently solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4a) ii. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.[4] (CCSS: 7.EE.4a) iii. Solve word problems[5] leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4b) iv. Graph the solution set of the inequality and interpret it in the context of the problem. (CCSS: 7.EE.4b)	**Inquiry Questions:** 1. Do algebraic properties work with numbers or just symbols? Why? 2. Why are there different ways to solve equations? 3. How are properties applied in other fields of study? 4. Why might estimation be better than an exact answer? 5. When might an estimate be the only possible answer? **Relevance and Application:** 1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed. 2. Comprehension of the structure of equations allows one to use spreadsheets effectively to solve problems that matter such as showing how long it takes to pay off debt, or representing data collected from science experiments. 3. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard. **Nature of Discipline:** 1. Mathematicians model with mathematics. (MP)

[2] Whole numbers, fractions, and decimals. (CCSS: 7.EE.3)
[3] For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2

	inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. (CCSS: 7.EE.3) [4] For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? (CCSS: 7.EE.4a) [5] For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions. (CCSS: 7.EE.4b)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.[1] (CCSS: 6.SP.1) b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS: 6.SP.2) c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.3) d. Summarize and describe distributions. (CCSS: 6.SP) i. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.4) ii. Summarize numerical data sets in relation to their context. (CCSS: 6.SP.5) 1. Report the number of observations. (CCSS: 6.SP.5a) 2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.5b) 3. Give quantitative measures of center (median and/or mean) and variability (inter-quartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.5c) 4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.5d)	**Inquiry Questions:** 1. Why are there so many ways to describe data? 2. When is one data display better than another? 3. When is one statistical measure better than another? 4. What makes a good statistical question? **Relevance and Application:** 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system. 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores. **Nature of Discipline:** 1. Mathematicians leverage strategic displays to reveal data. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP) [1] For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. (CCSS: 6.SP.1)

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

2. Statistics can be used to gain information about populations by examining samples.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: **a. Use random sampling to draw inferences about a population. (CCSS: 7.SP)** **i. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population.[1] (CCSS: 7.SP.1)** **ii. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.1)** **iii. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. (CCSS: 7.SP.2)** **iv. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (CCSS: 7.SP.2)** **b. Draw informal comparative inferences about two populations. (CCSS: 7.SP)** **i. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.[2] (CCSS: 7.SP.3)** **ii. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.[3] (CCSS: 7.SP.4)**	**Inquiry Questions:** **1. How might the sample for a survey affect the results of the survey?** **2. How do you distinguish between random and bias samples?** **3. How can you declare a winner in an election before counting all the ballots?** **Relevance and Application:** **1. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness or show strong opinions.** **2. Mathematical inferences allow us to make reliable predictions without accounting for every piece of data.** **Nature of Discipline:** 1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions. 2. Mathematicians use appropriate tools strategically. (MP) **[1] For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS: 7.SP.2)** **[2] For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS: 7.SP.3)** **[3] For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS: 7.SP.4)**

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

3. Mathematical models are used to determine probability.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: **a. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.[4] (CCSS: 7.SP.5)** **b. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.[5] (CCSS: 7.SP.6)** **c. Develop a probability model and use it to find probabilities of events. (CCSS: 7.SP.7)** **i. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.7)** **ii. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.[6] (CCSS: 7.SP.7a)** **iii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.[7] (CCSS: 7.SP.7b)** **d. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.8)** **i. Explain that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.8a)** **ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. (CCSS: 7.SP.8b)**	**Inquiry Questions:** 1. Why is it important to consider all of the possible outcomes of an event? 2. Is it possible to predict the future? How? 3. What are situations in which probability cannot be used? **Relevance and Application:** 1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers. 2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards. **Nature of Discipline:** 1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) **[4] Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.5)**

iii. **For an event[8] described in everyday language identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.8b)** iv. **Design and use a simulation to generate frequencies for compound events.[9] (CCSS: 7.SP.8c)**	[5] **For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS: 7.SP.6)** [6] **For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. (CCSS: 7.SP.7a)** [7] **For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS: 7.SP.7b)** [8] **e.g., "rolling double sixes" (CCSS: 7.SP.8b)** [9] **For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? (CCSS: 7.SP.8c)**

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

1. Objects in space and their parts and attributes can be measured and analyzed.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Develop and apply formulas and procedures for area of plane figures i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. (CCSS: 6.G.1) ii. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.1) b. Develop and apply formulas and procedures for volume of regular prisms. i. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. (CCSS: 6.G.2) ii. Show that volume is the same as multiplying the edge lengths of a rectangular prism. (CCSS: 6.G.2) iii. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.2) c. Draw polygons in the coordinate plan to solve real-world and mathematical problems. (CCSS: 6.G.3) i. Draw polygons in the coordinate plane given coordinates for the vertices. ii. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. (CCSS: 6.G.3) d. Develop and apply formulas and procedures for the surface area. i. Represent three-dimensional figures using nets made up of rectangles and triangles. (CCSS: 6.G.4)	**Inquiry Questions:** 1. Can two shapes have the same volume but different surface areas? Why? 2. Can two figures have the same surface area but different volumes? Why? 3. What does area tell you about a figure? 4. What properties affect the area of figures? **Relevance and Application:** 1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project. 2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes. **Nature of Discipline:** 1. Mathematicians realize that measurement always involves a certain degree of error. 2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians reason abstractly and quantitatively. (MP)

ii. Use nets to find the surface area of figures. (CCSS: 6.G.4) iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems. (CCSS: 6.G.4)	

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

2. Modeling geometric figures and relationships leads to informal spatial reasoning and proof.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: **a. Draw construct, and describe geometrical figures and describe the relationships between them. (CCSS: 7.G)** **i. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.1)** **ii. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. (CCSS: 7.G.2)** **iii. Construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.2)** **iv. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.3)**	**Inquiry Questions:** 1. Is there a geometric figure for any given set of attributes? 2. How does scale factor affect length, perimeter, angle measure, area and volume? 3. How do you know when a proportional relationship exists? **Relevance and Application:** 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography. 2. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Discipline:** 1. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 2. The relationship between geometric figures can be modeled 3. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 4. Mathematicians use appropriate tools strategically. (MP) 5. Mathematicians attend to precision. (MP)

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error.

GRADE LEVEL EXPECTATION: Pre Algebra

Concepts and skills students master:

3. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure.

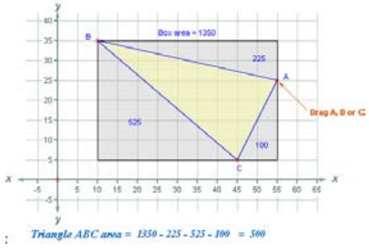
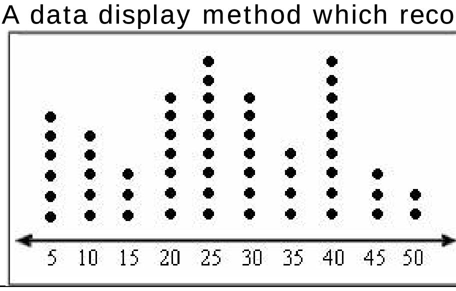
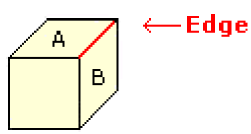
Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. **State the formulas for the area and circumference of a circle and use them to solve problems. (CCSS: 7.G.4)** b. **Give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.4)** c. **Use properties of supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.5)** d. **Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.6)**	**Inquiry Questions:** 1. **How can geometric relationships among lines and angles be generalized, described, and quantified?** 2. **How do line relationships affect angle relationships?** 3. **Can two shapes have the same volume but different surface areas? Why?** 4. **Can two shapes have the same surface area but different volumes? Why?** 5. **How are surface area and volume like and unlike each other?** 6. **What do surface area and volume tell about an object?** 7. **How are one-, two-, and three-dimensional units of measure related?** 8. **Why is pi an important number?**
	Relevance and Application: 1. **The ability to find volume and surface area helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use.**

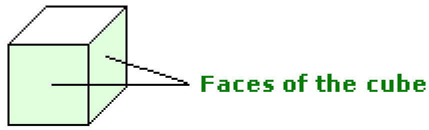
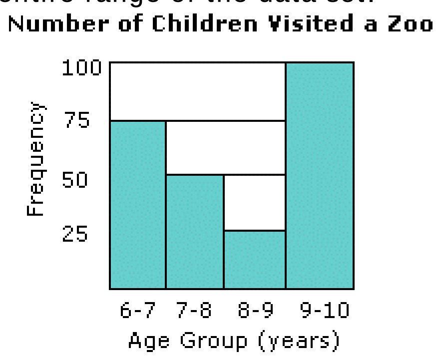
	Nature of Discipline: 1. Geometric objects are abstracted and simplified versions of physical objects. 2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

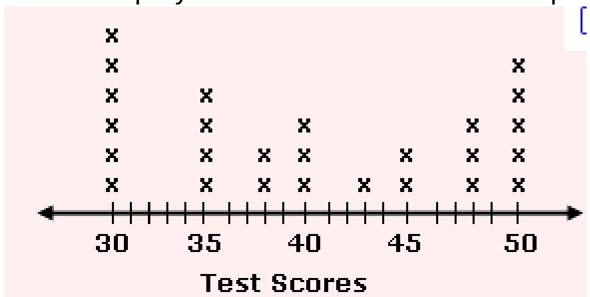
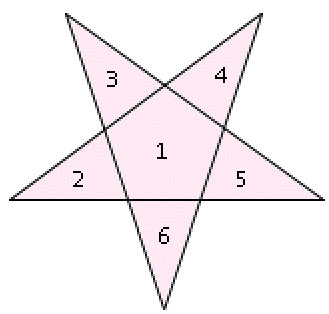
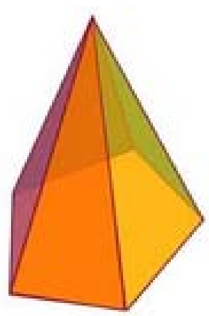
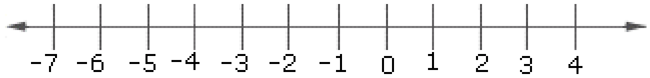
Glossary of Terms – M05

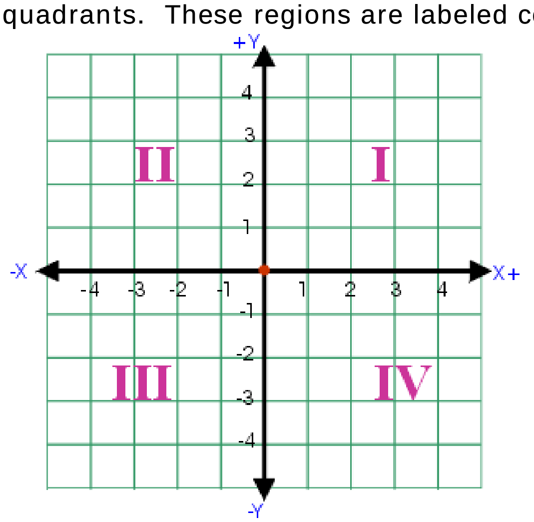
Academic Vocabulary
Standard 1: real-world problems problem-solving situations ratio, rates, unit rates, sum, product, quotient, difference, coordinate plane, percent, constant, tables, factor, common factor, GCF – greatest common factor, multiple, common multiple, LCM – least common multiple, distributive property, integers, parenthesis, permutations, positive numbers, prime number, negative numbers, opposites, number line, absolute value, rational numbers, reflection, inequality, quadrant, proper fraction, improper fraction, fraction, decimal, numerator, denominator, divisibility, equivalent, irrational numbers
Standard 2: algebraic methods, exponent, base, power, equation, expression, sum, term, product, factor, coefficient, quotient, variable, open sentence, optimization problems, order of operations, parenthesis, equivalent, equation, rational numbers, inequality, dependent variable, independent variable, quadrant function, linear function
Standard 3: data, distribution, statistical question, center, spread, mean, median, mode, range, histogram, box plot, dot plot, number line, data set, deviation, line plot, measures of central tendency, measures of variability, probability
Standard 4: congruent or the concept of congruence, coordinate geometry, formula, area, volume, triangles, polygons, quadrilaterals, surface area, rectangular prism, edge, coordinates, coordinate plane, three dimensional, nets, regular shapes, composite shapes, face

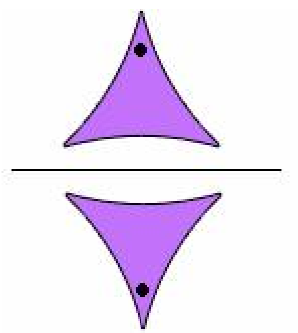
Word	Definition
Absolute value	A number's distance from zero on a number line. The absolute value of –6, shown as |6|, is 6, and the absolute value of 6, shown as |6|, is 6.
Algebraic methods	The use of symbols to represent numbers and signs to represent their relationships.
Algorithm	A step-by-step procedure.
Area	The number of square units a shape encloses.
Base number	The number in an exponential expression which is being multiplied by itself. (i.e. For example, 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)
Box plot (also called a box-and whiskers plot)	A graphic method for showing a summary of data using median, quartiles, and extremes of data. A box plot makes it easy to see where the data are spread out and where they are concentrated. The longer the box, the more the data are spread out.
Center	The middle point.
Coefficient	A number or constant quantity placed directly before the variable to indicate multiplication times the variable. (i.e. 2x means that 2 is the coefficient and represents 2 times "x".)
Common factors	Factors that two or more numbers share. (i.e. the numbers 2 and 8 share a common factor of 2.)
Common multiples	Multiples shared by two or more numbers. (i.e. the numbers 2 and 7 share a common multiple of 14.)
Composite shapes	Geometric figures that are created by combining two or more shapes. (i.e. a "house" created by a rectangle and a triangle)
Congruent or the concept of congruence	Two figures are said to be congruent if they are the same size and shape.
Constant	The part of a problem or equation that does not change.

Coordinate geometry	A system of geometry where algebra and the position of points on the standard x,y coordinate plane is used to study the properties of geometric figures. 
Coordinate Plane	A coordinate is a plane formed by the intersection of a horizontal number line with a vertical number line. The horizontal number line is called the *x*-axis and the vertical number line is called the *y*-axis. The number lines intersect at their zero points. This point of intersection is called the origin and written as (0, 0).
Coordinates	The ordered pairs on a coordinate plane identified by the position (x,y).
Conjecture	A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.
Data	Data can be defined as a collection of facts or information from which conclusions may be drawn.
Data set	A subset of the information that includes multiple pieces of data to be used for comparison or conclusion drawing.
Decimal	A decimal whose denominator is a power of ten and whose numerator is expressed by numerals placed to the right of the decimal point. (i.e. $^{56}/_{100}$ will be written as a decimal as .56)
Denominator	The bottom number in a fraction designating the number of parts in the whole.
Dependent variable	The variable in an expression whole value can vary depending on the value of the one or more independent variables. (i.e. In $p = 4q$, p is the dependent variable, because its value depends on the value of q.)
Deviation	The amount by which a number in the data set differs from a fixed value, such as the mean.
Difference	The solution to a subtraction problem.
Distribution	The way in which data is spread.
Distributive Property	The use of multiplication to “share” a number with two or more numbers in a sum. That is, $a(b + c) = ab + ac$. Formally, the Distributive Property states that “the product of a number and a sum is equal to the sum of the individual products of the addends and the number.”
Divisibility	The ability of one number to be divided into another number with a remainder of zero.
Dot plot	A data display method which records frequency using a “•” notation as shown below. 
Edge	The segment on a three-dimensional geometric figure that is formed by the intersection of two faces. 
Equation	A mathematical sentence that uses an equal sign to indicate two equal quantities.
Equivalent (fractions, expressions,	Equivalent Fractions are those fractions whose numerator and denominator are in the same ratio as that of the original fraction. Equivalent expressions are said to be equivalent if their values obtained by substituting the values of the variables are same.

equations)	Equivalent equations are the equations that have the same solution.
Exponent	A number used to tell how many times a number or variable is used as a factor. (i.e., 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)
Expression	A mathematical phrase containing numbers and/or variables and operation symbols. An expression is a representation of a value; for example, variables and/or numerals that appear alone or in combination with operators are expressions.
Face	A face is a flat surface of a three-dimensional figure. 
Factor	A whole number that divides another whole number with a remainder of zero.
Formula	A rule that shows the relationship between two or more quantities.
Fraction	A comparison of two numbers by division describing the part-to-whole relationship.
Greatest Common Factor, GCF	The largest factor shared by two or more numbers. (i.e. the numbers 75 and 100 have 5 and 25 as common factors between them. 25 is larger than 5, so 25 is the GCF for 75 and 100.)
Histogram	A bar graph that illustrates frequency over intervals. There are no spaces between the bars showing the frequency of data within each interval continuously over the entire range of the data set. 
Improper fraction	Fraction with a numerator greater than or equal to the denominator. Improper fractions can be simplified into a whole number and a remainder fraction.
Independent Variable	The variable in a situation that may have its value freely chosen regardless the values of any other variable. Changes in the independent variable may cause changes in the dependent variable.
Inequality	A mathematical sentence that contains a $<$, $>$, $\leq$, $\geq$, or $\neq$ symbol and shows that two values are not equal.
Integers	The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.
Irrational numbers	The set of numbers which cannot be represented as fractions. (i.e $\sqrt[3]{29}$, π, e, and $\sqrt{7}$).
Least Common Multiple, LCM	The smallest number that is a multiple of two or more numbers. (i.e. the numbers 5, 8 and 10 have common multiples of 40, 80, 120, etc. 40 is the smallest of the common multiples, so 40 is the LCM).
Linear function	A function that has a constant rate of change.

Line plot	A data display method which records frequency using “x” notation as shown below. 
Mean	The average of a set of numbers – the sum of the data divided by the number of the items.
Measures of central tendency	Numbers which in some sense communicate the “center” or “middle” of a set of data. The mean, median, and mode of statistical data are all measures of central tendency.
Measures of variability	Numbers which describe how spread out a set of data is, for example, range and quartile.
Median	The middle value when the data are arranged in numerical order.
Mental arithmetic	Performing computations in one’s head without writing anything down. Mental arithmetic strategies include finding pairs that add up to 10 or 100, doubling, halving and skip-counting.
Mode	The item in a data set that occurs with the greatest frequency.
Model	To make or construct a physical or mathematical representation.
Multiple	The product of the number and a non-zero whole number.
Negative numbers	Numbers less than zero.
Net	A two-dimensional pattern that can be folded to form a three dimensional figure. The following is the net of a pentagonal pyramid.  Folds to a 3-D Pentagonal  Pyramid.
Number line	A line on which numbers are marked at intervals. 
Number sense	An understanding of number. This would include number meanings, number relationships, number size, and the relative effect of operations on numbers.
Numerator	The top number in a fraction designating the number of parts.

Open sentence	A statement that contains at least one unknown. It becomes true or false when a quantity if substituted for the unknown. For example, 3 + x = 5
Opposites	Two numbers that are the same distance from zero on a number line, one positive, one negative.
Optimization problems	Real-world problems in which, given a number of constraints, the best solution is determined. For example, finding the best number of nonstop flights from Denver to San Francisco given the cost of fuel, number of passengers, number of crew required, etc.
Order of Operations	The rules to follow when more than one operation is present. Here are the rules: 1. Evaluate expressions inside parentheses. 2. Evaluate all powers. 3. Perform all multiplications and/or divisions from left to right. 4. Perform all additions and/or subtractions from left to right.
Parenthesis	A grouping symbol, ().
Pattern	Regularity in situations such as those in nature, events, shapes, designs, and sets of number (for example, spirals, on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12, …)
Percent	A ratio that compares a number to 100, commonly using the language “out of 100”. The symbol is %.
Permutations	All possible arrangements of a given number of items in which the order of the items makes a difference. For example, the different ways that a set of four books can be places on a shelf.
Polygon	A closed figure formed by three or more line segments that do not cross.
Positive number	Number greater than zero or to the right of zero on the number line.
Power	See Exponent. A number used to tell how many times a number or variable is used as a factor. (i.e., 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125. 5 is the base number and 3 is the exponent.)
Prime number	A counting number that can only be evenly divided by two different numbers, 1 and the number itself. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Probability	The likeliness or chance of an event occurring.
Problem-solving situations	Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.
Product	The answer to a multiplication problem.
Proper fraction	A fraction with a numerator that is less than the denominator.
Quadrant	The x and y axes of the coordinate plane divide the plane into four regions called quadrants. These regions are labeled counter-clockwise, starting from the top-right. 
Quadratic function	A function where the highest exponent on x is a 2. Standard form has an equation of the form $y = ax^2+bx+c$, where $a \neq 0$. (i.e. $y = 2x^2$ or $y = x^2 + 4x + 2$)
Quadrilaterals	A polygons with four sides.
Quotient	The answer to a division problem.

Range	The difference between the greatest and the least values in a data set.
Rate	A ratio that compares two quantities with different units.
Ratio	A comparison of two quantities by division.
Rational numbers	A number that can be expressed in the form *a/b*, where *a* and *b* are integers and b ≠0; for example, 3/4, 2/1, or 11/3. Every integer is a rational number, since it can be expressed in the form a/b, for example, 5 = 5/1. Rational numbers may be expressed as fractional or decimal numbers, for example, 3/4 or .75. Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.
Real numbers	All rational and irrational numbers.
Real-world problems	Quantitative problems that arise from a wide variety of human experience which may take into consideration contributions from various cultures (for example, Mayan or American pioneers), problems from abstract mathematics, and applications to various careers (for example, making change or calculating the sale price of an item). These may also be called real-world experiences, story problems, story contexts and word problems.
Rectangular prism	A three-dimensional shape formed by six rectangular faces.
Reflection	Is a transformation that flips a figure over a line of reflections. Also known as a flip. 
Regular shapes	Geometric figures that have the same measure for all side lengths and angle measures.
Spread	The range of a data set.
Statistical question	A question requiring data analysis to answer.
Sum	The answer to an addition problem.
Surface Area	The sum of all the areas of all the surfaces on a three dimensional figure.
Table	A type of organized data display. Used in statistics and algebra.
Term	A number in a pattern.
Three-dimensional	A figure that has length, width, and height.
Triangle	A polygon with three sides.
Unit Rate	The rate for one unit of a given quantity.
Variable	A letter that represents a number. The value of an algebraic expression varies, or changes, depending upon the value given to the variable. The solution to an algebraic equation is the value for the variable that makes the equation true.
Volume	The number of cubic units a three-dimensional figure encloses.

Definitions adapted from:

Boulder Valley School District Curriculum Essentials Document, 2009.

"Math Dictionary" www.icoachmath.com/math_dictionary/mathdictionarymain.html. Copyright © 1999 - 2011 HighPoints Learning Inc. December 30, 2011.

"Open Math Reference" www.openmathref.com. 2009 Copyright Math Open Reference. December 30, 2011.

"The Coordinate Plane." www.learningwave.com Copyright 1999 LearningWave Online, a division of HRM Video. December

"Thesaurus." http://www.thefreedictionary.com/statistical+distribution . Copyright © 2012 Farlex, Inc. January 5, 2012.

PK-12 Alignment of Mathematical Standards

The following pages will provide teachers with an understanding of the alignment of the standards from Pre-Kindergarten through High School. An understanding of this alignment and each grade level's role in assuring that each student graduates with a thorough understanding of the standards at each level is an important component of preparing our students for success in the 21st century. Provided in this section are the Prepared Graduate Competencies in Mathematics, an At-a-glance description of the Grade Level Expectations for each standard at each grade level, and a thorough explanation from the CCSS about the alignment of the standards across grade levels.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Mathematics
Prepared Graduate Competencies at Grade Levels
PK-12 Scope and Sequence

Understand the structure and properties of our number system. At the most basic level numbers are abstract symbols that represent real-world quantities.		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.1-GLE.1	The complex number system includes real numbers and imaginary numbers
Eighth Grade	MA10-GR.8-S.1-GLE.1	In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
Sixth Grade	MA10-GR.6-S.1-GLE.3	In the real number system, rational numbers have a unique location on the number line and in space
Fifth Grade	MA10-GR.5-S.1-GLE.1	The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms
	MA10-GR.5-S.1-GLE.4	The concepts of multiplication and division can be applied to multiply and divide fractions
Fourth Grade	MA10-GR.4-S.1-GLE.1	The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms
Third Grade	MA10-GR.3-S.1-GLE.1	The whole number system describes place value relationships and forms the foundation for efficient algorithms
Second Grade	MA10-GR.2-S.1-GLE.1	The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms
First Grade	MA10-GR.1-S.1-GLE.1	The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms
Kindergarten	MA10-GR.K-S.1-GLE.1	Whole numbers can be used to name, count, represent, and order quantity

Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error.		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.1-GLE.2	Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
Seventh Grade	MA10-GR.7-S.4-GLE.2	Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure
Fifth Grade	MA10-GR.5-S.4-GLE.1	Properties of multiplication and addition provide the foundation for volume an attribute of solids.
Fourth Grade	MA10-GR.4-S.4-GLE.1	Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time
Third Grade	MA10-GR.3-S.4-GLE.2	Linear and area measurement are fundamentally different and require different units of measure
	MA10-GR.3-S.4-GLE.3	Time and attributes of objects can be measured with appropriate tools
Second Grade	MA10-GR.2-S.4-GLE.2	Some attributes of objects are measurable and can be quantified using different tools
First Grade	MA10-GR.1-S.4-GLE.2	Measurement is used to compare and order objects and events
Kindergarten	MA10-GR.K-S.4-GLE.2	Measurement is used to compare and order objects
Preschool	MA10-GR.P-S.1-GLE.1	Quantities can be represented and counted
	MA10-GR.P-S.4-GLE.2	Measurement is used to compare objects

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.4	Solutions to equations, inequalities and systems of equations are found using a variety of tools
Eight Grade	MA10-GR.8-S.2-GLE.2	Properties of algebra and equality are used to solve linear equations and systems of equations
Seventh Grade	MA10-GR.7-S.1-GLE.2	Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
Sixth Grade	MA10-GR.6-S.1-GLE.2	Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency
Fifth Grade	MA10-GR.5-S.1-GLE.2	Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency
	MA10-GR.5-S.1-GLE.3	Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency
Fourth Grade	MA10-GR.4-S.1-GLE.3	Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency
Third Grade	MA10-GR.3-S.1-GLE.3	Multiplication and division are inverse operations and can be modeled in a variety of ways
Second Grade	MA10-GR.2-S.1-GLE.2	Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.		
Grade Level	**Numbering System**	**Grade Level Expectations**
Seventh Grade	MA10-GR.7-S.1-GLE.1	Proportional reasoning involves comparisons and multiplicative relationships among ratios
Sixth Grade	MA10-GR.6-S.1-GLE.1	Quantities can be expressed and compared using ratios and rates

Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.3-GLE.3	Probability models outcomes for situations in which there is inherent randomness
Seventh Grade	MA10-GR.7-S.3-GLE.2	Mathematical models are used to determine probability

Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.3-GLE.1	Visual displays and summary statistics condense the information in data sets into usable knowledge
Eighth Grade	MA10-GR.8-S.3-GLE.1	Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
Sixth Grade	MA10-GR.6-S.3-GLE.1	Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge
Fifth Grade	MA10-GR.5-S.3-GLE.1	Visual displays are used to interpret data
Fourth Grade	MA10-GR.4-S.3-GLE.1	Visual displays are used to represent data
Third Grade	MA10-GR.3-S.3-GLE.1	Visual displays are used to describe data
Second Grade	MA10-GR.2-S.3-GLE.1	Visual displays of data can be constructed in a variety of formats to solve problems
First Grade	MA10-GR.1-S.3-GLE.1	Visual displays of information can used to answer questions

Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.3	Expressions can be represented in multiple, equivalent forms
High School	MA10-GR.HS-S.2-GLE.1	Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically
Seventh Grade	MA10-GR.7-S.2-GLE.1	Properties of arithmetic can be used to generate equivalent expressions
Fourth Grade	MA10-GR.4-S.1-GLE.2	Different models and representations can be used to compare fractional parts
Third Grade	MA10-GR.3-S.1-GLE.2	Parts of a whole can be modeled and represented in different ways

Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.1	Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables

Fifth Grade	MA10-GR.5-S.2-GLE.1	Number patterns are based on operations and relationships
Fourth Grade	MA10-GR.4-S.2-GLE.1	Number patterns and relationships can be represented by symbols
Preschool	MA10-GR.P-S.4-GLE.1	Shapes can be observed in the world and described in relation to one another

Apply transformation to numbers, shapes, functional representations, and data

Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.4-GLE.1	Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically
High School	MA10-GR.HS-S.4-GLE.3	Objects in the plane can be described and analyzed algebraically
Eighth Grade	MA10-GR.8-S.4-GLE.1	Transformations of objects can be used to define the concepts of congruence and similarity
Seventh Grade	MA10-GR.7-S.4-GLE.1	Modeling geometric figures and relationships leads to informal spatial reasoning and proof
Second Grade	MA10-GR.2-S.4-GLE.1	Shapes can be described by their attributes and used to represent part/whole relationships
First Grade	MA10-GR.1-S.1-GLE.2	Number relationships can be used to solve addition and subtraction problems
Kindergarten	MA10-GR.K-S.1-GLE.2	Composing and decomposing quantity forms the foundation for addition and subtraction

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level	Numbering System	Grade Level Expectations
High School	MA10-GR.HS-S.4-GLE.4	Attributes of two- and three-dimensional objects are measurable and can be quantified
Sixth Grade	MA10-GR.6-S.2-GLE.1	Algebraic expressions can be used to generalize properties of arithmetic
	MA10-GR.6-S.2-GLE.2	Variables are used to represent unknown quantities within equations and inequalities
	MA10-GR.6-S.4-GLE.1	Objects in space and their parts and attributes can be measured and analyzed
Fifth Grade	MA10-GR.5-S.4-GLE.2	Geometric figures can be described by their attributes and specific locations in the plane
Fourth Grade	MA10-GR.4-S.4-GLE.2	Geometric figures in the plane and in space are described and analyzed by their attributes
Third Grade	MA10-GR.3-S.4-GLE.1	Geometric figures are described by their attributes
First Grade	MA10-GR.1-S.4-GLE.1	Shapes can be described by defining attributes and created by composing and decomposing
Kindergarten	MA10-GR.K-S.4-GLE.1	Shapes can be described by characteristics and position and created by composing and decomposing

Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

This prepared graduate competency is addressed through all of the grade level expectations and is part of the mathematical practices.

Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions		
Grade Level	**Numbering System**	**Grade Level Expectations**
High School	MA10-GR.HS-S.2-GLE.2	Quantitative relationships in the real world can be modeled and solved using functions
	MA10-GR.HS-S.3-GLE.2	Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions
	MA10-GR.HS-S.4-GLE.2	Concepts of similarity are foundational to geometry and its applications
	MA10-GR.HS-S.4-GLE.5	Objects in the real world can be modeled using geometric concepts
Eighth Grade	MA10-GR.8-S.2-GLE.3	Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
	MA10-GR.8-S.4-GLE.2	Direct and indirect measurement can be used to describe and make comparisons
Seventh Grade	MA10-GR.7-S.2-GLE.2	Equations and expressions model quantitative relationships and phenomena
	MA10-GR.7-S.3-GLE.1	Statistics can be used to gain information about populations by examining samples

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
High School	
1. Number Sense, Properties, and Operations	2. The complex number system includes real numbers and imaginary numbers 3. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
2. Patterns, Functions, and Algebraic Structures	4. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 5. Quantitative relationships in the real world can be modeled and solved using functions 6. Expressions can be represented in multiple, equivalent forms 7. Solutions to equations, inequalities and systems of equations are found using a variety of tools
3. Data Analysis, Statistics, and Probability	3. Visual displays and summary statistics condense the information in data sets into usable knowledge 4. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions 5. Probability models outcomes for situations in which there is inherent randomness
4. Shape, Dimension, and Geometric Relationships	5. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically 6. Concepts of similarity are foundational to geometry and its applications 7. Objects in the plane can be described and analyzed algebraically 8. Attributes of two- and three-dimensional objects are measurable and can be quantified 9. Objects in the real world can be modeled using geometric concepts

From the Common State Standards for Mathematics, Pages 58, 62, 67, 72-74, and 79.

Mathematics | High School—Number and Quantity
Numbers and Number Systems. *During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.*

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ *should be* $5^{(1/3)3} = 5^1 = 5$ *and that* $5^{1/3}$ *should be the cube root of 5.*

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. *In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.*

Mathematics | High School—Algebra

Expressions. *An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.*

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. *An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.*

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. *Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.*

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- *Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.*
- *Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.*
- *Designing the layout of the stalls in a school fair so as to raise as much money as possible.*
- *Analyzing stopping distance for a car.*
- *Modeling savings account balance, bacterial colony growth, or investment growth.*
- *Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.*
- *Analyzing risk in situations such as extreme sports, pandemics, and terrorism.*
- *Relating population statistics to individual predictions.*

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

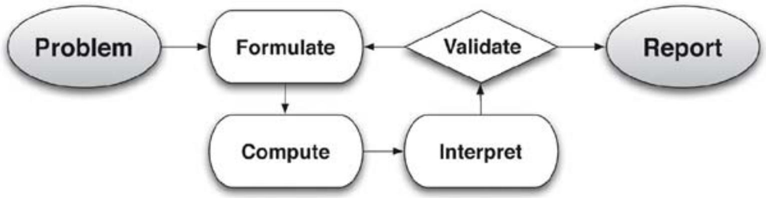
The basic modeling cycle is summarized in the diagram (below). It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards. *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*



Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two *possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.*

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. *The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.*

*Mathematics | High School—Statistics and Probability**

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling*. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.*

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Eighth Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically 2. Properties of algebra and equality are used to solve linear equations and systems of equations 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity 2. Direct and indirect measurement can be used to describe and make comparisons

From the Common State Standards for Mathematics, Page 52.

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and

graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Seventh Grade	
1. Number Sense, Properties, and Operations	1. Proportional reasoning involves comparisons and multiplicative relationships among ratios 2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
2. Patterns, Functions, and Algebraic Structures	1. Properties of arithmetic can be used to generate equivalent expressions 2. Equations and expressions model quantitative relationships and phenomena
3. Data Analysis, Statistics, and Probability	1. Statistics can be used to gain information about populations by examining samples 2. Mathematical models are used to determine probability
4. Shape, Dimension, and Geometric Relationships	1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof 2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

From the Common State Standards for Mathematics, Page 46.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve
a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on

congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Sixth Grade	
1. Number Sense, Properties, and Operations	1. Quantities can be expressed and compared using ratios and rates 2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency 3. In the real number system, rational numbers have a unique location on the number line and in space
2. Patterns, Functions, and Algebraic Structures	1. Algebraic expressions can be used to generalize properties of arithmetic 2. Variables are used to represent unknown quantities within equations and inequalities
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Objects in space and their parts and attributes can be measured and analyzed

<u>From the Common State Standards for Mathematics, Pages 39-40</u>

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve

simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios,
and they use equations (such as 3x = y) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Fifth Grade	
1. Number Sense, Properties, and Operations	1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms 2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency 3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency 4. The concepts of multiplication and division can be applied to multiply and divide fractions
2. Patterns, Functions, and Algebraic Structures	1. Number patterns are based on operations and relationships
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to interpret data
4. Shape, Dimension, and Geometric Relationships	1. Properties of multiplication and addition provide the foundation for volume an attribute of solids 2. Geometric figures can be described by their attributes and specific locations in the plane

From the Common State Standards for Mathematics, Page 33.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Fourth Grade	
1. Number Sense, Properties, and Operations	1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms 2. Different models and representations can be used to compare fractional parts 3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency
2. Patterns, Functions, and Algebraic Structures	1. Number patterns and relationships can be represented by symbols
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to represent data
4. Shape, Dimension, and Geometric Relationships	1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time 2. Geometric figures in the plane and in space are described and analyzed by their attributes

From the Common State Standards for Mathematics, Page 27.

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Third Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships and forms the foundation for efficient algorithms 2. Parts of a whole can be modeled and represented in different ways 3. Multiplication and division are inverse operations and can be modeled in a variety of ways
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to describe data
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures are described by their attributes 2. Linear and area measurement are fundamentally different and require different units of measure 3. Time and attributes of objects can be measured with appropriate tools

From the Common State Standards for Mathematics, Page 21.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Second Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms 2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays of data can be constructed in a variety of formats to solve problems
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be described by their attributes and used to represent part/whole relationships 2. Some attributes of objects are measurable and can be quantified using different tools

From the Common State Standards for Mathematics, Page 17.

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
First Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms 2. Number relationships can be used to solve addition and subtraction problems
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays of information can be used to answer questions
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be described by defining attributes and created by composing and decomposing 2. Measurement is used to compare and order objects and events

From the Common State Standards for Mathematics, Page 13.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry

[1]*Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.*

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Kindergarten	
1. Number Sense, Properties, and Operations	1. Whole numbers can be used to name, count, represent, and order quantity 2. Composing and decomposing quantity forms the foundation for addition and subtraction
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes are described by their characteristics and position and created by composing and decomposing 2. Measurement is used to compare and order objects

From the Common State Standards for Mathematics, Page 9.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Preschool	
1. Number Sense, Properties, and Operations	1. Quantities can be represented and counted
2. Patterns, Functions, and Algebraic Structures	1. Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be observed in the world and described in relation to one another 2. Measurement is used to compare objects