

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Standard 1: GLE 1. The complex number system includes real numbers and imaginary numbers					
Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
a. Extend the properties of exponents to rational exponents. (CCSS: N-RN)					
i.	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. ¹ (CCSS: N-RN.1)	M25	Algebra 1	Algebra 2	The concept of rational exponents is introduced in M25. ¹ For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. (CCSS: N-RN.1)
ii.	Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2)	Algebra 1	Algebra 1	Algebra 2	
b. Use properties of rational and irrational numbers. (CCSS: N-RN)					
i.	Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3)	Idea of closed set - Algebra 1	Algebra 1	Algebra 2	
ii.	Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3)	Idea of closed set - Algebra 1	Algebra 1	Algebra 2	
iii.	Explain why the product of a nonzero rational number and an irrational number is irrational. (CCSS: N-RN.3)	Algebra 1	Algebra 1	Algebra 2	
c. Perform arithmetic operations with complex numbers. (CCSS: N-CN)					
i.	Define the complex number i such that $i^2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real numbers. (CCSS: N-CN.1)	Algebra 2	Algebra 2	PreCalculus & Calculus	

Boulder Valley School District
Level of Mastery Document
High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
ii.	Use the relation $i^2=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: N-CN.2)	Algebra 2	Algebra 2	PreCalculus & Calculus	
	d. Use complex numbers in polynomial identities and equations. (CCSS: N-CN)				
i.	Solve quadratic equations with real coefficients that have complex solutions. (CCSS: N-CN.7)	Algebra 2	Algebra 2	Algebra 2	
Standard 1. GLE 2: Quantitative reasoning is used to make sense of quantities and their relationship in problem situations.					
	a. Reason quantitatively and use units to solve problems (CCSS: N-Q)				
i.	Use units as a way to understand problems and to guide the solution of multi-step problems. (CCSS: N-Q.1)	Introduced in 5th grade	Algebra 1	On-going	
1	Choose and interpret units consistently in formulas. (CCSS: N-Q.1)	Begins in M05	Algebra 1	On-going	
2	Choose and interpret the scale and the origin in graphs and data displays. (CCSS: N-Q.1)	M25	Algebra 1	On-going	
ii.	Define appropriate quantities for the purpose of descriptive modeling. (CCSS: N-Q.2)	Algebra 1	Algebra 1	On-going	
iii	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: N-Q.3)	M25	Algebra 1	On-going	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iv.	Describe factors affecting take-home pay and calculate the impact (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
v.	Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
Standard 2. GLE 1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables					
	a. Formulate the concept of a function and use function notation. (CCSS: F-IF)				
i.	Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. ¹ (CCSS: F-IF.1)	M25	Algebra 1	Algebra 2	¹ If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$. (CCSS: F-IF.1)
ii.	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: F-IF.2)	M25	Algebra 1	Algebra 2	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Demonstrate that sequences are functions, ² sometimes defined recursively, whose domain is a subset of the integers. (CCSS: F-IF.3)	Algebra 1	Algebra 2	Algebra 2	² For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. (CCSS: F-IF.3)
b. Interpret functions that arise in applications in terms of the context. (CCSS: F-IF)					
i.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features ³ given a verbal description of the relationship. (CCSS: F-IF.4)	M15: Linear; Algebra 1: exponential and quadratic. Algebra 2: Emphasize selection of appropriate functions for use in problem solving situations.	M25: Linear; Algebra 1: Linear & Exponential; Algebra 2: exponential and quadratic	Algebra 2: quadratic, exponential, piecewise, step; PreCalculus: logarithmic	³ Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS: F-IF.4)
ii.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ⁴ (CCSS: F-IF.5)	M25: linear	Algebra 1: linear, exponential and quadratic. Algebra 2: Emphasize selection of appropriate functions for use in problem solving situations.		⁴ For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS: F-IF.5)

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Calculate and interpret the average rate of change ⁵ of a function over a specified interval. Estimate the rate of change from a graph. (CCSS: F-IF.6)	M05: concept of rate of change and using tables; M15: tables, graphs and basic linear equations; M25: Linear; Algebra 1: Exponential and quadratic	Algebra 2: Exponential and quadratics, Emphasize selection of appropriate functions for use in problem solving situations. Rate of change, represented symbolically or as a table.	M25: linear; Algebra 2: quadratic, exponential, piecewise, step; PreCalculus: logarithmic	⁵ presented symbolically or as a table. (CCSS: F-IF.6)
c. Analyze functions using different representations. (CCSS: F-IF)					
i.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (CCSS: F-IF.7)	Algebra 1: linear, quadratic, exponential; Algebra 2: all functions	Algebra 1: linear, quadratic, exponential; Algebra 2: all functions	On-going	
ii.	Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: F-IF.7a)	M25: Linear; Algebra 1: quadratic	Algebra 1: linear; Algebra 2: quadratic	PreCalculus: All functions	

Boulder Valley School District
Level of Mastery Document
High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: F-IF.7b)	Algebra 1: introduction of all function types, define peicewise; Algebra 2: Able to graph by hand, focus on using key features of functions for selecting appropriate function for problem-solving	Algebra 2: Able to graph by hand, focus on using key features of functions for selecting appropriate function for problem-solving	PreCalculus	
iv.	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: F-IF.7c)	Algebra 2	Algebra 2	PreCalculus: all functions	
v.	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: F-IF.7e)	Algebra 1: Exponential Functions; Algebra 2: logarithmic and trigonometric functions	Algebra 2	Algebra 2 & PreCalculus	
vi.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8)	Algebra 1: sums, differences, factoring quadratics;	Algebra 2: factoring beyond quadratics	Algebra 2 & PreCalculus	

Boulder Valley School District
Level of Mastery Document
High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
1	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: F-IF.8a)	Algebra 1	Algebra 2	Pre-Calculus	
2	Use the properties of exponents to interpret expressions for exponential functions. (CCSS: F-IF.8b)	Algebra 1	Algebra 2	Algebra 2 & PreCalculus	
3	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (CCSS: F-IF.9)	Algebra 1: linear, quadratic, exponential; Algebra 2: logarithmic, trigonometric, piecewise, step	Algebra 2: linear, quadratic, polynomial, exponential; PreCalculus: logarithmic, trigonometric, piecewise, step	Algebra 2 & PreCalculus	
	d. Build a function that models a relationship between two quantities. (CCSS: F-BF)				
i.	Write a function that describes a relationship between two quantities. (CCSS: F-BF.1)	M25: linear; Algebra 1: quadratic, exponential; Algebra 2: logarithmic	Algebra 1: linear; Algebra 2: quadratic, polynomial, exponential; PreCalculus: logarithmic	PreCalculus	
1	Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: F-BF.1a)	Algebra 1	Algebra 1	Algebra 2	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
2	Combine standard function types using arithmetic operations. ⁸ (CCSS: F-BF.1b)	Algebra 1: linear, quadratic, exponential; Algebra 2: polynomial	Algebra 1: linear, quadratic, exponential; Algebra 2: polynomial	Algebra 2 & PreCalculus	⁸ For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (CCSS: F-BF.1b)
ii.	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (CCSS: F-BF.2)	Algebra 1	Algebra 1	Advanced Algebra 2; PreCalculus	
e. Build new functions from existing functions. (CCSS: F-BF)					
i.	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k , ⁹ and find the value of k given the graphs. ¹⁰ (CCSS: F-BF.3)	Algebra 1: linear, quadratic, exponential; Algebra 2: polynomial	Algebra 2	Algebra 2 & PreCalculus	⁹ both positive and negative. (CCSS: F-BF.3) ¹⁰ Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS: F-BF.3)
ii.	Experiment with cases and illustrate an explanation of the effects on the graph using technology.	Teaching strategy for transformations (CCSS:F-BF.3)	Teaching strategy for transformations (CCSS:F-BF.3)	Teaching strategy for transformations	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Find inverse functions. ¹¹ (CCSS: F-BF.4)	Algebra 1: Finding $f^{-1}(y)=x$. Linear only. Algebra 2: Finding inverse for all functions. PreCalculus: Use the composition to verify. Find inverse using restricted domain. Use tables to find inverse functions.	Algebra 1: Finding $f^{-1}(y)=x$. Linear only. Algebra 2: Finding inverse for all functions. PreCalculus: Use the composition to verify. Find inverse using restricted domain. Use tables to find inverse functions.	Algebra 2 & PreCalculus	¹¹ Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
f.	Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF)	Algebra 2	Trigonometry semester of PreCalculus	Calculus	
i.	Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1)	Algebra 2	PreCalculus	Calculus	
ii.	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)	Algebra 2	PreCalculus	Calculus	
Standard 2. GLE 2: Quantitative relationships in the real world can be modeled and solved using functions					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
	a. Construct and compare linear, quadratic, and exponential models and solve problems. (CCSS: F-LE)				
i.	Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: F-LE.1)	M25	Algebra 1	Algebra 2	
1	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: F-LE.1a)	M25	Algebra 1	Algebra 2	
2	Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: F-LE.1b)	M25	Algebra 1	Algebra 2	
3	Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: F-LE.1c)	Algebra 1	Algebra 1	Algebra 2 & PreCalculus	
ii.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs. ¹² (CCSS: F-LE.2)	Algebra 1; Algebra 2:natural log and base e	Algebra 1: linear; Algebra 2: exponential, logarithmic	Algebra 1: linear; Algebra 2:address natural logarithms, exponential with $y=ae^{kt}$; PreCalculus: as a dampening factor	¹² include reading these from a table. (CCSS: F-LE.2)

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: F-LE.3)	Algebra 1	Algebra 2	Algebra 2 & PreCalculus	
iv.	For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology. (CCSS: F-LE.4)	Algebra 2	Algebra 2	PreCalculus	
b. Interpret expressions for function in terms of the situation they model. (CCSS: F-LE)					
i.	Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: F-LE.5)	Algebra 1	Algebra 2	Algebra 1: linear; Algebra 2: exponential	
c. Model periodic phenomena with trigonometric functions. (CCSS: F-TF)					
i.	Choose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. (CCSS: F-TF.5)	Algebra 2	PreCalculus	PreCalculus	
d. Model personal financial situations					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Analyze the impact of interest rates on a personal financial plan (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
ii.	Evaluate the costs and benefits of credit (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
iii.	Analyze various lending sources, services, and financial institutions (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
Standard 2. GLE 3: Expressions can be represented in multiple, equivalent forms					
	a. Interpret the structure of expressions.(CCSS: A-SSE)				

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Interpret expressions that represent a quantity in terms of its context. (CCSS: A-SSE.1)	M05: Using variables as unknowns; M15: Expressions with variables as a part of equations; M25: applying equations in a linear context; Algebra 1: linear, exponential, quadratic; Algebra 2: polynomial, rational	Algebra 1: linear, exponential; Algebra 2: quadratic; PreCalculus: polynomial, rational	Algebra 2 & PreCalculus	
1	Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: A-SSE.1a)	M05	M25	Algebra 1 and up	
2	Interpret complicated expressions by viewing one or more of their parts as a single entity. ¹³ (CCSS: A-SSE.1b)	M05	Algebra 1: linear, exponential, quadratic; Algebra 2: polynomial, rational	Algebra 1 and up	¹³ For example, interpret $P(1+r)^t$ as the product of P and a factor not depending on P. (CCSS: A-SSE.1b)
ii.	Use the structure of an expression to identify ways to rewrite it. ¹⁴ (CCSS: A-SSE.2)	Algebra 1: linear, exponential, quadratic; Algebra 2: polynomial, rational	Algebra 1: linear, exponential, quadratic; Algebra 2: polynomial, rational	Algebra 2: all	¹⁴ For example, see $x^2 - y^2$ as $(x^2) - (y^2)$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (CCSS: A-SSE.2)

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
b. Write expressions in equivalent forms to solve problems. (CCSS: A-SSE)					
i.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (CCSS: A-SSE.3)	Algebra 1: linear, exponential, quadratic	Algebra 1: factoring quadratics, writing different forms of linear equations	Algebra 1 and up	
1	Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a)	Algebra 1	Algebra 2	Algebra 2 & PreCalculus	
2	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b)	Algebra 1	Algebra 2	Algebra 2 & PreCalculus	
3	Use the properties of exponents to transform expressions for exponential functions. ¹⁵ (CCSS: A-SSE.3c)	Algebra 1	Algebra 2	Algebra 2 & PreCalculus	¹⁵ For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (CCSS: A-SSE.3c)
ii.	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. (CCSS: A-SSE.4)	Algebra 2	Algebra 2	Calculus	
c. Perform arithmetic operations on polynomials. (CCSS: A-APR)					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: A-APR.1)	Algebra 1: linear, quadratic; Algebra 2: all functions beyond quadratic	Algebra 1: linear, quadratic; Algebra 2: all functions beyond quadratic	PreCalculus	
	d. Understand the relationship between zeros and factors of polynomials. (CCSS: A-APR)				
i.	State and apply the Remainder Theorem. ¹⁷ (CCSS: A-APR.2)	Algebra 2: all functions beyond quadratic	Algebra 2: all functions beyond quadratic	PreCalculus	¹⁷ For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. (CCSS: A-APR.2)
ii.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: A-APR.3)	Algebra 1:quadratic; Algebra 2: all functions beyond quadratic	Algebra 1:quadratic; Algebra 2: all functions beyond quadratic	PreCalculus	
	e. Use polynomial identities to solve problems. (CCSS: A-APR)				
i.	Prove polynomial identities ¹⁸ and use them to describe numerical relationships. (CCSS: A-APR.4)	Algebra 2: all functions beyond quadratic	Algebra 2	Algebra 2 & PreCalculus	¹⁸ For example, the polynomial identity $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ can be used to generate Pythagorean triples. (CCSS: A-APR.4)
	f. Rewrite rational expressions with linear and quadratic denominators. (CCSS: A-APR)	Algebra 2	Algebra 2	Algebra 2 & PreCalculus	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
	g. Rewrite simple rational expressions in different forms. ¹⁹ (CCSS: A-APR.6)	Algebra 2	Algebra 2	Algebra 2 & PreCalculus	¹⁹ write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$ where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: A-APR.6)
Standard 2. GLE 4: Solutions to equations, inequalities and systems of equations are found using a variety of tools					
	a. Create equations that describe numbers or relationships. (CCSS: A-CED)				
i.	Create equations and inequalities ²⁰ in one variable and use them to solve problems. (CCSS: A-CED.1)	M15	Algebra 1	Algebra 2 & PreCalculus	²⁰ Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (CCSS: A-CED.1)
ii.	Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: A-CED.2)	M25	Algebra 1	Algebra 2 & PreCalculus	
iii.	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. ²¹ (CCSS: A-CED.3)	Algebra 1	Algebra 1	Algebra 2	²¹ For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (CCSS: A-CED.3)
iv.	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ²² (CCSS: A-CED.4)	M15	M25	Algebra 1 and up	²² For example, rearrange Ohm's law $V = IR$ to highlight resistance R . (CCSS: A-CED.4)
	b. Understand solving equations as a process of reasoning and explain the reasoning. (CCSS: A-REI)				

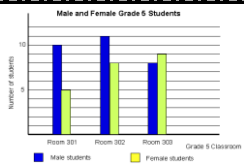
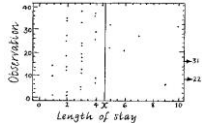
Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. (CCSS: A-REI.1)	M15	M25	Algebra 1 and up	The concept, for example, that subtracting 2 from both sides of $2x + 2 = 12$, maintains an equivalent relationship.
ii.	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: A-REI.2)	Algebra 2: simple radical, rational	Algebra 2	Algebra 2 & PreCalculus	
c. Solve equations and inequalities in one variable. (CCSS: A-REI)					
i.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS: A-REI.3)	M15: one-step, no variable coefficient; M25: two-step with variable coefficient.	M25	Algebra 1 and up	
ii.	Solve quadratic equations in one variable. (CCSS: A-REI.4)	Algebra 1	Algebra 1	Algebra 2 & PreCalculus	
1	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = K$ that has the same solutions. Derive the quadratic formula from this form. (CCSS: A-REI.4a)	Algebra 1: linear and quadratics with real solutions	Algebra 2: using ax^2 and deriving the quadratic formula.	Algebra 2 & PreCalculus	
2	Solve quadratic equations ²³ by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. (CCSS: A-REI.4b)	Algebra 1: linear and quadratics with real solutions	Algebra 2	Algebra 2 and up	²³ e.g., for $x^2=49$. (CCSS: A-REI.4b)

Boulder Valley School District
Level of Mastery Document
High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
3	Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b . (CCSS: A-REI.4b)	Algebra 2	Algebra 2	Algebra 2 and up	
d. Solve systems of equations. (CCSS: A-REI)					
i.	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: A-REI.5)	Algebra 1: for combinations:linear and linear or linear and quadratic	Algebra 1: for combinations:linear and linear or linear and quadratic	Algebra 2 and up	
ii.	Solve systems of linear equations exactly and approximately, ²⁴ focusing on pairs of linear equations in two variables. (CCSS: A-REI.6)	Algebra 1	Algebra 1	Algebra 2 and up	²⁴ e.g., with graphs. (CCSS: A-REI.6)
iii.	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. ²⁵ (CCSS: A-REI.7)	Algebra 1	Algebra 1	Algebra 2 and up	²⁵ For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (CCSS: A-REI.7)
e. Represent and solve equations and inequalities graphically. (CCSS: A-REI)					
i.	Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve. ²⁶ (CCSS: A-REI.10)	M25	Algebra 1	Algebra 2 and up	²⁶ which could be a line. (CCSS: A-REI.10)

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
ii.	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; ²⁷ find the solutions approximately. ²⁸ * (CCSS: A-REI.11)	Algebra 1: linear, exponential as general principle; Algebra 2: combine, polynomial, radical, rational, absolute value and exponential	Algebra 1: linear, exponential as general principle; Algebra 2: combine, polynomial, radical, rational, absolute value and exponential	Algebra 2 and up	²⁷ Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. (CCSS: A-REI.11) ²⁸ e.g., using technology to graph the functions, make tables of values, or find successive approximations. (CCSS: A-REI.11)
iii.	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: A-REI.12)	Algebra 1	Algebra 1	Algebra 2	
Standard 3. GLE 1: Visual displays and summary statistics condense the information in data sets into usable knowledge					
a. Summarize, represent, and interpret data on a single count or measurement variable. (CCSS: S-ID)					
i.	Represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS: S-ID.1)	M05: all forms of data displays	M05: use and application	Algebra 1 & up	 <p>Histogram: Dot Plot:</p> 

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
ii.	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: S-ID.2)	M15	Algebra 1	Algebra 2	
iii.	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: S-ID.3)	M05	M15	Algebra 1 & up	
iv.	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. (CCSS: S-ID.4)	Algebra 2	Algebra 2	Statistics	
v.	Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: S-ID.4)	Algebra 2	Algebra 2	PreCalculus & Calculus	
	b. Summarize, represent, and interpret data on two categorical and quantitative variables. (CCSS: S-ID)				
i.	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: S-ID.5)	Algebra 2	Algebra 2	Statistics	
ii.	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: S-ID.6)	Algebra 1	Algebra 1	Algebra 2 and up	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
1	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: S-ID.6a)	Algebra I	Algebra 1	Algebra 2	
2	Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: S-ID.6b)	Algebra I	Algebra 1	Algebra 2	
3	Fit a linear function for a scatter plot that suggests a linear association. (CCSS: S-ID.6c)	Algebra I	Algebra 1	Algebra 2, Statistics	
c. Interpret linear models. (CCSS: S-ID)					
i.	Interpret the slope ² and the intercept ³ of a linear model in the context of the data. (CCSS: S-ID.7)	Algebra I	Algebra 1	Algebra 2	² rate of change. (CCSS: S-ID.7) ³ constant term. (CCSS: S-ID.7)
ii.	Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: S-ID.8)	Algebra I	Algebra 1	Algebra 2	
iii.	Distinguish between correlation and causation. (CCSS: S-ID.9)	Algebra I	Algebra 2	Statistics	
Standard 3. GLE 2: Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions					
a. Understand and evaluate random processes underlying statistical experiments. (CCSS: S-IC)					
i.	Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: S-IC.1)	Algebra 2	Algebra 2	Statistics	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
ii.	Decide if a specified model is consistent with results from a given data-generating process. ⁴ (CCSS: S-IC.2)	Algebra 2	Algebra 2	Statistics	⁴ e.g., using simulation. (CCSS: S-IC.2) For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (CCSS: S-IC.2)
b. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (CCSS: S-IC)					
i.	Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3)	Algebra 2	Algebra 2	Statistics	
ii.	Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4)	Algebra 2	Algebra 2	Statistics	
iii.	Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4)	Algebra 2	Algebra 2	Statistics	
iv.	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5)	Algebra 2	Algebra 2	Statistics	
v.	Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size).	Algebra 2	Algebra 2	Statistics	
vi.	Evaluate reports based on data. (CCSS: S-IC.6)	Algebra 2	Algebra 2	Algebra 2 and up	
Standard 3. GLE 3: Probability models outcomes for situations in which there is inherent randomness					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
	a. Understand independence and conditional probability and use them to interpret data. (CCSS: S-CP)				
i.	Describe events as subsets of a sample space ⁵ using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events. ⁶ (CCSS: S-CP.1)	Geometry	Geometry	Statistics, Discrete	⁵ the set of outcomes. (CCSS: S-CP.1) ⁶ "or," "and," "not". (CCSS: S-CP.1)
ii.	Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: S-CP.2)	Geometry	Geometry	Statistics, Discrete	
	Using the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS: S-CP.3)	Geometry	Geometry	Statistics, Discrete	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ⁷ (CCSS: S-CP.4)	Geometry	Geometry	Statistics, Discrete	⁷ For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (CCSS: S-CP.4)
v.	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. ⁸ (CCSS: S-CP.5)	Geometry	Geometry	Statistics, Discrete	⁸ For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (CCSS: S-CP.5)
	b. Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP)				
i.	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS: S-CP.6)	Geometry	Geometry	Statistics, Discrete	
ii.	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (CCSS: S-CP.7)	Geometry	Geometry	Statistics, Discrete	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
	c. Analyze the cost of insurance as a method to offset the risk of a situation. (PFL)	Personal Finance or Money Management Online Course	Personal Finance or Money Management Online Course	On-going, in context	Due to the loose nature of the timing for completion of the graduation required online course, students may or may not have been exposed to this material prior to their senior year.
Standard 4. GLE 1: Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically					
	a. Experiment with transformations in the plane. (CCSS: G-CO)				
i.	State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: G-CO.1)	Geometry	Geometry	Algebra 2 and up, in context	
ii.	Represent transformations in the plane using ¹ appropriate tools. (CCSS: G-CO.2)	Geometry	Geometry	Algebra 2 and up, in context	¹ e.g., transparencies and geometry software. (CCSS: G-CO.2)
iii.	Describe transformations as functions that take points in the plane as inputs and give other points as outputs. (CCSS: G-CO.2)	Geometry	Geometry	Algebra 2 and up, in context	
iv.	Compare transformations that preserve distance and angle to those that do not. ² (CCSS: G-CO.2)	Geometry	Geometry	Algebra 2 and up, in context	² e.g., translation versus horizontal stretch. (CCSS: G-CO.2)
v.	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS: G-CO.3)	Geometry	Geometry	Algebra 2 and up, in context	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
vi.	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: G-CO.4)	Geometry	Geometry	Algebra 2 and up, in context	
vii.	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools. ³ (CCSS: G-CO.5)	Geometry	Geometry	Algebra 2 and up, in context	³ e.g., graph paper, tracing paper, or geometry software. (CCSS: G-CO.5)
viii.	Specify a sequence of transformations that will carry a given figure onto another. (CCSS: G-CO.5)	Geometry	Geometry	Algebra 2 and up, in context	
b. Understand congruence in terms of rigid motions. (CCSS: G-CO)					
i.	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. (CCSS: G-CO.6)	Geometry	Geometry	Algebra 2 and up, in context	
ii.	Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: G-CO.6)	Geometry	Geometry	Algebra 2 and up, in context	
iii.	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: G-CO.7)	Geometry	Geometry	Algebra 2 and up, in context	
iv.	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: G-CO.8)	Geometry	Geometry	Algebra 2 and up, in context	
c. Prove geometric theorems. (CCSS: G-CO)					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Prove theorems about lines and angles. ⁴ (CCSS: G-CO.9)	Geometry	Geometry	Geometry	Logic is maintained in both Discrete and Problem-Solving, though not in the context of Geometric proof.
ii.	Prove theorems about triangles. ⁵ (CCSS: G-CO.10)	Geometry	Geometry	Geometry	⁴ Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (CCSS: G-CO.9)
iii.	Prove theorems about parallelograms. ⁶ (CCSS: G-CO.11)	Geometry	Geometry	Geometry	⁵ Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (CCSS: G-CO.10)
d. Make geometric constructions. (CCSS: G-CO)			⁶ Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (CCSS: G-CO.11)		

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Make formal geometric constructions ⁷ with a variety of tools and methods. ⁸ (CCSS: G-CO.12)	Geometry	Geometry	Geometry	⁷ Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (CCSS: G-CO.12) ⁸ compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. (CCSS: G-CO.12)
ii.	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: G-CO.13)	Geometry	Geometry	Geometry	
Standard 4. GLE 2: Concepts of similarity are foundational to geometry and its applications					
	a. Understand similarity in terms of similarity transformations. (CCSS: G-SRT)				
i.	Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: G-SRT.1)	Geometry	Geometry		
1	Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: G-SRT.1a)	Geometry	Geometry	Calculus	

Boulder Valley School District
Level of Mastery Document
High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
2	Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: G-SRT.1b)	Geometry	Geometry		
ii.	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. (CCSS: G-SRT.2)	Geometry	Geometry	Geometry	
iii.	Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: G-SRT.2)	Geometry	Geometry	Geometry	
iv.	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: G-SRT.3)	Geometry	Geometry	Geometry	
	b. Prove theorems involving similarity. (CCSS: G-SRT)				
i.	Prove theorems about triangles. ⁹ (CCSS: G-SRT.4)	Geometry	Geometry	Geometry	⁹ Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (CCSS: G-SRT.4)
ii.	Prove that all circles are similar. (CCSS: G-C.1)	Geometry	Geometry	PreCalculus, in the context of conics	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
iii.	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: G-SRT.5)	Geometry	Geometry	Geometry	
c. Define trigonometric ratios and solve problems involving right triangles. (CCSS: G-SRT)					
i.	Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: G-SRT.6)	Geometry	Geometry	Algebra 2 and up, in context	
ii.	Explain and use the relationship between the sine and cosine of complementary angles. (CCSS: G-SRT.7)	Geometry	Geometry	Algebra 2 and up, in context	
iii.	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (CCSS: G-SRT.8)	Geometry	Geometry	Algebra 2 and up, in context	
d. Prove and apply trigonometric identities. (CCSS: F-TF)					
i.	Prove the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. (CCSS: F-TF.8)	Geometry, Pythagorean identity only	PreCalculus/Trigonometry	Trigonometry and up	
ii.	Use the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (CCSS: F-TF.8)	Geometry	PreCalculus/Trigonometry	Trigonometry and up	
e. Understand and apply theorems about circles. (CCSS: G-C)					

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
i.	Identify and describe relationships among inscribed angles, radii, and chords. ¹⁰ (CCSS: G-C.2)	Geometry	Geometry	Algebra 2 and up, in context	¹⁰ Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (CCSS: G-C.2)
ii.	Construct the inscribed and circumscribed circles of a triangle. (CCSS: G-C.3)	Geometry	Geometry	Algebra 2 and up, in context	
iii.	Prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: G-C.3)	Geometry	Geometry	Algebra 2 and up, in context	
f. Find arc lengths and areas of sectors of circles. (CCSS: G-C)					
i.	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality. (CCSS: G-C.5)	Geometry	Geometry		
ii.	Derive the formula for the area of a sector. (CCSS: G-C.5)	Geometry	Geometry		
Standard 4. GLE 3: Objects in the plane can be described and analyzed algebraically					
a. Express Geometric Properties with Equations. (CCSS: G-GPE)					
i.	Translate between the geometric description and the equation for a conic section. (CCSS: G-GPE)	Geometry	Geometry	Algebra 2 and up	

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem. (CCSS: G-GPE.1)	Geometry	Geometry	Trigonometry and up	
2	Complete the square to find the center and radius of a circle given by an equation. (CCSS: G-GPE.1)	Geometry	Geometry	PreCalculus	
3	Derive the equation of a parabola given a focus and directrix. (CCSS: G-GPE.2)	PreCalculus	PreCalculus	Calculus	
ii.	Use coordinates to prove simple geometric theorems algebraically. (CCSS: G-GPE)	Geometry	Geometry	Algebra 2	
1	Use coordinates to prove simple geometric theorems ¹¹ algebraically. (CCSS: G-GPE.4)	Geometry	Geometry	Algebra 2	¹¹ For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$. (CCSS: G-GPE.4)
2	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. ¹² (CCSS: G-GPE.5)	Geometry	Geometry	Algebra 2 and up	¹² e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (CCSS: G-GPE.5)
3	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: G-GPE.6)	Geometry	Geometry		

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
4	Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles. (CCSS: G-GPE.7)	Geometry	Geometry	Algebra 2	
4. Attributes of two- and three-dimensional objects are measurable and can be quantified					
	a. Explain volume formulas and use them to solve problems. (CCSS: G-GMD)				Geometry - Use formulas, Pre Calc - derive, explain and use formulas
i.	Give an informal argument ¹³ for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. (CCSS: G-GMD.1)	Geometry	Geometry	Geometry	¹³ Use dissection arguments, Cavalieri's principle and informal limit arguments. (CCSS: G-GMD.1)
ii.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (CCSS: G-GMD.3)	M25	Geometry	Calculus	
	b. Visualize relationships between two-dimensional and three-dimensional objects. (CCSS: G-GMD)				
i.	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: G-GMD.4)	Geometry	Geometry	Calculus	
5. Objects in the real world can be modeled using geometric concepts					
	a. Apply geometric concepts in modeling situations (CCSS:G-MG)				
i.	Use geometric shapes, their measures, and their properties to describe objects. ¹⁴ (CCSS: G-MG.1)	Geometry	Geometry	Geometry	¹⁴ e.g., modeling a tree trunk or a human torso as a cylinder. (CCSS: G-MG.1)

Boulder Valley School District
 Level of Mastery Document
 High School Mathematics

Evidence Outcomes		INTRODUCE	MASTER	MAINTAIN	Notes
ii.	Apply concepts of density based on area and volume in modeling situations. ¹⁵ (CCSS: G-MG.2)	Geometry	Geometry	Calculus 2	¹⁵ e.g., persons per square mile, BTUs per cubic foot. (CCSS: G-MG.2)
iii.	Apply geometric methods to solve design problems. ¹⁶ (CCSS: G-MG.3)	Geometry	Geometry	Algebra 2 and up	¹⁶ e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems