

BVSD Second Grade Mathematics Transition Overview

Moving from 2009 BVSD Math Standards (2009 CED) to New Colorado Academic Standards for Mathematics (CAS) and the Boulder Valley Curriculum Essentials - 2012

Introduction: The new Colorado Academic Standards represent a major shift in mathematics education. These standards, based off of the Common Core State Standards for Mathematics (CCSSM), which have been adopted by more than 45 states, are the result of a national movement to align our standards with the highest performing nations in the world. Boulder Valley has adopted the Colorado Academic Standards as our Curriculum Essentials verbatim.

Central to the new standards are the 8 Standards for Mathematical Practice. These standards align with the previous NCTM Standards for Mathematical Processes which were included in the 2009 BVSD Math Standards as the last Essential Learning at each grade level. The chart below illustrates how the Practice Standards align with the Process standards.

Process Standards and Practice Standards

New Standards for Mathematical Practice	Expectations from 2000 NCTM Standards and BVSD 2009 Math Standards
1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically	Problem Solving <ul style="list-style-type: none"> • Selects, applies, and translates among mathematical representations to solve problems and justifies the reasonableness of solutions • Given a real-world problems selects an appropriate method to solve the problem by determining if the information provided is sufficient, insufficient or extraneous • Creates and illustrates a real-world problem from a given math sentence
2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning.	Reasoning and Proof <ul style="list-style-type: none"> • Analyzes and evaluates the mathematical thinking, strategies and arguments of peers, teachers and others
3. Construct viable arguments.	Communication <ul style="list-style-type: none"> • Develops, tests and explains mathematical conjectures • Recognizes and utilizes key academic vocabulary relevant to mathematics in verbal and written communication
6. Attend to precision. 7. Look for and make use of structure.	Connections <ul style="list-style-type: none"> • Recognizes, uses, and explains connections among mathematical ideas in contexts both inside and outside of mathematics classrooms
4. Model with mathematics.	Representations <ul style="list-style-type: none"> • Creates and uses representations to organize, record and explain mathematical ideas clearly to peers, teachers and others

State Standards: There has also been a shift at the level of the content standards. In the prior Colorado State Standards there were six standards. These have been condensed into four standards. Standards 1 and 6 have been combined as have standards 4 and 5. This shift had been anticipated and incorporated into the 2009 CED. The chart below illustrates this alignment.

2010 Colorado Academic Standards	2005 Colorado Model Content Standards
Standard 1: Number Sense, Properties, and Operations	Standard 1: Number Sense and Number Relationships Standard 6: Computation
Standard 2: Patterns, Functions, and Algebraic Structures	Standard 2: Patterns and Algebra
Standard 3: Data Analysis, Statistics, and Probability	Standard 3: Data and Probability
Standard 4: Shape, Dimension and Geometric Relationships	Standard 4: Geometry Standard 5: Measurement

Personal and Financial Literacy (PFL): Standards related to PFL are integrated throughout the new standards. Although these are not a part of the Common Core State Standards, Colorado added these standards at each grade level when adopting new standards at the State level.

Second Grade Overview

Standard 1: Number Sense, Properties, and Operations

Standard 1 represents more than half of the second grade curriculum. Some of the ideas align perfectly with the 2009 BVSD Math Standards. Knowing addition and subtraction facts under 20, and fluency with addition and subtraction under 100 remain central to the work in 2nd grade. Students will have opportunities to solve addition and subtraction problems with numbers up to 1000 (supported by materials and drawings). Work related to fractions of arrays will continue to be a part of second grade expectations.

Standard 2: Patterns, Functions, and Algebraic Structure

The ideas related to Standard 2 are integrated into the other standards. There are no specific Evidence Outcomes relating to Standard 2 in the 2010 CAS. Growth patterns are no longer included in the second grade curriculum.

Standard 3: Data Analysis, Statistics, and Probability

The CAS greatly limits the work related to data analysis and probability in second grade by focusing on recording measurement data, and only specifies using line plots, bar graphs and picture graphs. Probability has been eliminated from 2nd grade.

Standard 4: Shape, Dimension, and Geometric Relationship

Much of the work related to shapes and polygons will remain unchanged in the new curriculum. Right angles are no longer mentioned in second grade. Symmetry is also not mentioned in the new standards.

Measurement in second grade will focus exclusively on measurement of length. Students will be expected to measure to the nearest inch or centimeter. Explicit connections of linear measurements to number lines, and solving addition and subtraction problems within those contexts are central to this standard. Reading a clock to the nearest 5 minutes remains the same.

Colorado Academic Standards (CAS) Alignment to BVSD 2009 Math Standards (2009 CED)

Purpose: This document has been created to help teachers understand how the new standards for mathematics (CAS) align with the previous standards: BVSD 2009 Math Standards (2009 CED). Often there is a partial alignment of the new with the old, for that reason the commentary in the notes column was included to help teachers to understand the shift. We hope that this will facilitate teams in the revision of their curriculum maps, assessments, and instructional units.

Second Grade

Standard: 1. Number Sense, Properties, and Operations		
Concepts and Skills: 1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms		
CAS Evidence Outcomes	Alignment with 2009 CED	Notes
<p>a. Use place value to read, write, count, compare, and represent numbers. (CCSS: 2.NBT)</p> <p>i. Represent the digits of a three-digit number as hundreds, tens, and ones.¹ (CCSS: 2.NBT.1)</p> <p>ii. Count within 1000. (CCSS: 2.NBT.2)</p> <p>iii. Skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.2)</p> <p>iv. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.3)</p> <p>v. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 2.NBT.4)</p>	<ul style="list-style-type: none"> • 2M1a: Reads, writes, compares, and orders whole numbers from 0-1000 including the use of symbolic notation (e.g., $<$, $>$, $=$) • 2M1b: Says the number word sequence: By 1s: forward to 1000 and backward from 100 starting from any number in the sequence; By 2s and 10s: forward and backward between 0-100 starting from any number; By 5s: forward and backward between 0-100 starting from any multiple of 5; By 100s: forward and backward on the century 0-1000 • 2M1d: Creates equivalent representations of whole numbers (e.g., 35 can be represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) 	<ul style="list-style-type: none"> • Although not explicitly mentioned in the standards, we consider “Count within 1000” to include both forward and backward counting, starting at any number. • CAS specifically mentions writing number names with words, which is new. • CAS includes skip counting by 2 with the topic of odd and even (2M1.2.d.i.)

¹ e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.1) 100 can be thought of as a bundle of ten tens — called a “hundred.” (CCSS: 2.NBT.1a) The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.1b)

Standard: 1. Number Sense, Properties, and Operations

Concepts and Skills: 1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms (cont'd)

CAS Evidence Outcomes	Alignment with 2009 CED	Notes
<p>b. Use place value understanding and properties of operations to add and subtract. (CCSS: 2.NBT)</p> <p>i. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.5)</p> <p>ii. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.6)</p> <p>iii. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.² (CCSS: 2.NBT.7)</p> <p>iv. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.8)</p> <p>v. Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS: 2.NBT.9)</p>	<ul style="list-style-type: none">• 2M2b: Solves two-digit addition and subtraction situations to 100 using mental and written base-ten strategies and explains them using models and representation• 2M2c: Uses part-part-whole knowledge of quantity to model and solve related addition and subtraction problems (e.g. $19+3 = 22$; $22 - 3 = 19$; $22 - 19 = 3$)• 2M2e: Uses commutative and associative properties to solve multi-digit addition problems• 3M1g: Models and solves addition and subtraction problems to 1000 using efficient, accurate, flexible, and generalizable methods and writes number sentences to fit a given situation• 2M2f: Adds and subtracts multiples of 100 in the range of 1 to 1000• 3M1a: Says the number sequence using 1s, 2s, 10s, and 100s, forward to 10,000 and backward from 1000 starting from any number	<ul style="list-style-type: none">• Addition of up to four two digit numbers is new to 2nd grade.• CAS distinguishes between computational fluency within 100 and the use of drawings, models or other strategies to add and subtract within 1000.• CAS specifies mentally adding or subtracting 10 or 100 to numbers within 1000.• Estimation of a quantity of objects in a set, and estimation of sums and differences are not included in CAS (2009 CED 2M1c, 2M2g).

² Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.7)

Standard: 1. Number Sense, Properties, and Operations		
Concepts and Skills: 2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency		
CAS Evidence Outcomes	Alignment with 2009 CED	Notes
a. Represent and solve problems involving addition and subtraction. (CCSS: 2.OA) <ul style="list-style-type: none"> i. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.³ (CCSS: 2.OA.1) ii. Apply addition and subtraction concepts to financial decision-making (PFL) 	<ul style="list-style-type: none"> • 2M2b: Solves two-digit addition and subtraction situations to 100 using mental and written base-ten strategies and explains them using models and representation • 2M2d: Models situations and solves problems involving missing addends and subtrahends with whole numbers to 100, emphasizing the concept of equivalency • 2M1d Number (Money) Connection: Creates equivalent combinations for coins to \$1.00 (e.g., a quarter equals five nickels or two dimes and one nickel) 	<ul style="list-style-type: none"> • The 2009 CED did not specify two-step problems.
b. Fluently add and subtract within 20 using mental strategies. (CCSS: 2.OA.2)	<ul style="list-style-type: none"> • 2M2a: Uses efficient mental strategies to calculate addition and subtraction facts to 20 	<ul style="list-style-type: none"> • CAS expectations align with 2009 CED.
c. Know from memory all sums of two one-digit numbers. (CCSS: 2.OA.2)	<ul style="list-style-type: none"> • 2M2a: Uses efficient mental strategies to calculate addition and subtraction facts to 20 	<ul style="list-style-type: none"> • CAS expectations align with 2009 CED.

³ e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.OA.1)

Standard: 1. Number Sense, Properties, and Operations

Concepts and Skills: 2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency (cont'd)

CAS Evidence Outcomes	Alignment with 2009 CED	Notes
<p>d. Use equal groups of objects to gain foundations for multiplication. (CCSS: 2.OA)</p> <ul style="list-style-type: none">i. Determine whether a group of objects (up to 20) has an odd or even number of members.⁴ (CCSS: 2.OA.3)ii. Write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.3)iii. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.4)	<ul style="list-style-type: none">• 1M1h: Puts objects in a collection into equal groups and uses the group structure to count• 1M1h Algebra Connection: Identifies collections of objects as either odd or even• 2M1b: Geometry Connection: Arranges objects in arrays and determines quantity by skip or stress counting• 2M4b: Can construct and identify rows and columns in rectangular arrays	<ul style="list-style-type: none">• Expressing even numbers as the sum of 2 equal addends is new. (e.g. $14 = 7 + 7$)

⁴ e.g., by pairing objects or counting them by 2s. (CCSS: 2.OA.3)

Standard: 2. Patterns, Functions, and Algebraic Structures

Concepts and Skills:

CAS Evidence Outcomes	Alignment with 2009 CED	Notes
"Expectations for this standard are integrated into the other standards at this grade level." - Colorado Academic Standards, (CAS)		<ul style="list-style-type: none">• Topics related to growing patterns are not included in CAS at 2nd grade.<ul style="list-style-type: none">○ 2M1b Algebra Connection: Recognizes and models repeating and growing patterns (e.g., two eyes for one head) and uses them to solve related problems

Standard: 3. Data Analysis, Statistics, and Probability

Concepts and Skills: 1. Visual displays of data can be constructed in a variety of formats to solve problems

CAS Evidence Outcomes	Alignment with 2009 CED	Notes
<p>a. Represent and interpret data. (CCSS: 2.MD)</p> <p>i. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.9)</p> <p>ii. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. (CCSS: 2.MD.10)</p> <p>iii. Solve simple put together, take-apart, and compare problems using information presented in picture and bar graphs. (CCSS: 2.MD.10)</p>	<ul style="list-style-type: none">• 2M3b: Collects data using observation, counting, and surveys• 2M3c: Creates a concrete or pictorial representation of data (e.g., tallies, pictographs, line plots, bar graphs, frequency tables, and Venn diagrams)• 2M3d: Interprets and makes accurate statements about a representation of data including identifying the mode and shape of the data	<ul style="list-style-type: none">• CAS limits data representations to line plots, picture graphs and bar graphs. It also limits data to 4 categories.• CAS makes specific mention of collecting linear measurement data to be recorded on a line plot.• Students are not expected to determine mode in CAS.• CAS is specific about solving problems to interpret data.• Probability (most likely, least likely etc.) is not included in CAS. (2009 CED 2M3d)

Standard: 4. Shape, Dimension, and Geometric Relationships		
Concepts and Skills: 1. Shapes can be described by their attributes and used to represent part/whole relationships		
CAS Evidence Outcomes	Alignment with 2009 CED	Notes
a. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (CCSS: 2.G.1)	<ul style="list-style-type: none"> • 1M3c: Identifies, describes, and compares shapes (rectangles, circles, triangles, hexagon, rhombus, trapezoid) using common attributes (e.g., number of sides) and identifies the shapes that are the faces of solid figures • 2M4e: Identifies and classifies rectangles and triangles based on the number of sides, angles, and right angles. 	<ul style="list-style-type: none"> • CAS expects students to draw shapes. • CAS includes pentagons. • Right angles are not specified in the CAS.
b. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.1)	<ul style="list-style-type: none"> • 1M3c: Identifies, describes, and compares shapes (rectangles, circles, triangles, hexagon, rhombus, trapezoid) using common attributes (e.g., number of sides) and identifies the shapes that are the faces of solid figures • 2M4e: Identifies and classifies rectangles and triangles based on the number of sides, angles, and right angles. 	<ul style="list-style-type: none"> • CAS expectations align with 2009 CED. • Right angles are not specified in the CAS.
c. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.2)	<ul style="list-style-type: none"> • 2M4a: Constructs rectangular arrays to represent and compare numbers • 2M1b: Geometry Connection: Arranges objects in arrays and determines quantity by skip or stress counting 	<ul style="list-style-type: none"> • CAS expectations align with 2009 CED.
d. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. (CCSS: 2.G.3)	<ul style="list-style-type: none"> • 2M4c: Identify halves, thirds, and fourths of rectangular arrays • 2M4c Number Connection: Reads, writes, and identifies thirds, fourths, and halves • 2M4c Measurement Connection: Relates halves and fourths to half and quarter hours on a clock 	<ul style="list-style-type: none"> • Partitioning circles is new. • 2009 CED did not mention describing the whole as the sum of its fractional parts. • Relating halves to symmetry is not included in CAS (2009 CED 2M4d)
e. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.3)	<ul style="list-style-type: none"> • 3M2e: Models, names, and compares common fractions (halves, thirds, fourths) as equal partitions of a whole or set and locates on a number line (e.g., place $3\frac{1}{2}$ on a number line) 	<ul style="list-style-type: none"> • CAS specifies that equal share do not have to have the same shape.

Standard: 4. Shape, Dimension, and Geometric Relationships

Concepts and Skills: 2. Some attributes of objects are measurable and can be quantified using different tools

CAS Evidence Outcomes	Alignment with 2009 CED	Notes
<p>a. Measure and estimate lengths in standard units. (CCSS: 2.MD)</p> <p>i. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.1)</p> <p>ii. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.2)</p> <p>iii. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.3)</p> <p>iv. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.4)</p>	<ul style="list-style-type: none">• 2M3b Measurement Connection: Uses centimeters and inches to collect quantifiable data related to length• 3M3e: Estimates the measurements of familiar objects using appropriate standard units (e.g., a paper clip is about one inch; a pencil is about 10 centimeters)	<ul style="list-style-type: none">• All measurement in 2nd grade CAS is limited to length.• Using different units to measure the same object and comparing results is new.• Finding differences in lengths connects to addition and subtraction comparison problems.

Standard: 4. Shape, Dimension, and Geometric Relationships

Concepts and Skills: 2. Some attributes of objects are measurable and can be quantified using different tools (Cont'd)

<p>b. Relate addition and subtraction to length. (CCSS: 2.MD)</p> <p>i. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units¹ and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.5)</p> <p>ii. Represent whole numbers as lengths from 0 on a number line² diagram and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.6)</p>	<ul style="list-style-type: none"> • 2M2b: Solves two-digit addition and subtraction situations to 100 using mental and written base-ten strategies and explains them using models and representation. • 2M3d Algebra Connection: Describes qualitative change (e.g., student is growing taller) and quantitative change (e.g., the student grew 2 inches) 	<ul style="list-style-type: none"> • CAS is specific about using number line diagrams and making the connection to measurement. • Symbols to represent unknown numbers may include letters, question marks, or geometric figures (e.g. $42 - \square = 15$) • ii. is new.
<p>c. Solve problems time and money. (CCSS: 2.MD)</p> <p>i. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.7)</p> <p>ii. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately.³ (CCSS: 2.MD.8)</p>	<ul style="list-style-type: none"> • 2M1b Measurement Connection: Tells/reads time to nearest five minutes • 2M1d Number (Money) Connection: Creates equivalent combinations for coins to \$1.00 (e.g., a quarter equals five nickels or two dimes and one nickel). 	<ul style="list-style-type: none"> • CAS expectations for time align with 2009 CED • CAS is specific about solving word problems involving money and includes dollar bills.

¹ e.g., by using drawings (such as drawings of rulers). (CCSS: 2.MD.5)

² with equally spaced points corresponding to the numbers 0, 1, 2, ... (CCSS: 2.MD.6)

³ Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS: 2.MD.6)