A global at math
The topic of fractions is an important concept in elementary school mathematics. The study of fractions not only enables students to perform computations but also provides a foundation for later work with rates, percents, slope, and other topics in secondary school mathematics. However, research has long reported that many students and even adults have difficulty understanding operations with fractions (Behr et al. 1992). Similar results have been observed in international assessments. For instance, a recent report from the Trends in International Mathematics and Science Study (TIMSS) shows that U.S. students still do not perform as well as their counterparts in many other countries, especially Asian countries, including Japan, South Korea, and Singapore (Mullis et al. 2008).

As a lens to more clearly view mathematics instruction in the United States, many researchers have focused on features of curricula or textbooks in these high-achieving countries. Among important studies to note on the topic of fractions are the Watanabe articles (2001, 2006) published in Teaching Children Mathematics, which examine the initial treatment of fraction concepts in Japanese elementary school math textbooks. Pointing out the later introduction of fractions in Japan, Watanabe suggests that formal instruction of fractions in the United States
be delayed from the third to the fourth grade. Moreover, he suggests using linear and liquid measurement as introductory contexts for fractions in U.S. textbooks, highlighting the prevalence in Japanese mathematics textbooks of linear measurement contexts (e.g., liter, number line), the limited use of area models (e.g., pizza), and the complete lack of discrete model use (e.g., counters). Whereas Watanabe's suggestions help North American instructors reflect on their own practices, at the same time, such comments raise the question of how fractions are taught in high-achieving Asian countries.

Korean students start formal schooling at the same age that American students do and go through grades 1–12. Korean students' hours in school range from 7 to 10 hours a day, 6 days a week (sometimes 5 days) for about 220 days a year. The Korean school system is similar to Japan's; both countries have centralized systems that follow a 6-3-3 ladder pattern, which consists of elementary school (grades 1–6), middle school (grades 7–9), and high school (grades 10–12).

Korean students consistently ranked second highest in mathematics achievement in a variety of international studies, including the 1999 and 2007 TIMSS studies. However, despite their outstanding performance in international assessments, little research exists that addresses how Korean students learn mathematics, especially the challenging topic of fractions, apropos of the textbook organization.

The purpose of this study is not to judge the quality of Korean, Japanese, or U.S. approaches to fraction instruction but rather to use the practices of another country to examine our own practices in order to find a better way to support student learning.

**Korean mathematics curriculum**

A nationwide program of study developed by the Ministry of Education, the Korean mathematics curriculum has been changed seven times since public education began in Korea in the 1950s. **Table 1** summarizes the treatment of fractions in the current curriculum, called the *7th Korean Curriculum*, in compari-

---

**Table 1**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Korean curriculum</th>
<th>Focal points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3</strong></td>
<td>• Introduction of fractions; equal-sharing (parts of a whole); unit and nonunit fractions</td>
<td>Introduce fractions and have students understand the meanings and uses of fractions to represent parts of a whole, parts of a set, and points or distances on a number line; comparing and ordering of fractions; fraction equivalence</td>
</tr>
<tr>
<td></td>
<td>• Fractions as numbers in discrete quantities (parts of a set); relationships between unit fractions and nonunit fractions; comparison of fractions with like denominators; introduction of decimal numbers in relation to fractions; comparison of decimals</td>
<td></td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>• Various fractions including improper fractions and mixed numbers; comparison of fractions and mixed numbers; addition and subtraction with like denominators</td>
<td>Develop understanding of decimals, including the connections between fractions and decimals</td>
</tr>
<tr>
<td></td>
<td>• Fractions as ratio and as quotient; fractions equal to one; addition and subtraction of fractions with like denominators and whole numbers or mixed numbers</td>
<td></td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>• Addition and subtraction of nonfractions with unlike denominators; addition and subtraction of mixed numbers and among three fractions with unlike denominators; multiplication of fractions</td>
<td>Develop understanding of and fluency with addition and subtraction; develop fluency with standard procedures for adding and subtracting fractions and decimals</td>
</tr>
<tr>
<td></td>
<td>• Division of fractions; multiplication and division of decimals</td>
<td></td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>• Relationship between fractions and decimal numbers</td>
<td>Develop understanding of and fluency with multiplying and dividing fractions and decimals</td>
</tr>
<tr>
<td></td>
<td>• Division of fractions with unlike denominators and of improper fractions; division of mixed numbers</td>
<td></td>
</tr>
</tbody>
</table>
son to the treatment of fractions at the same grade level suggested in *Curriculum Focal Points* (NCTM 2006), which provides descriptions of the most significant mathematical concepts and skills to be developed at each grade level in the U.S. curriculum. Table 1 shows the recommendation that the concept of fractions be introduced at the same time, in grade 3, in both circumstances. However, more careful analysis shows that in most cases, fraction concepts in the Korean curriculum are introduced and developed at a slightly earlier stage. For example, decimal numbers are introduced in grade 3 in the Korean curriculum, but NCTM recommends they be introduced in grade 4. Moreover, although the multiplication of fractions is introduced and developed intensively in grade 5 in the Korean curriculum, *Curriculum Focal Points* (NCTM 2006) recommends them for grade 6. In general, topics discussed at each grade level in the Korean curriculum seem to be much more mathematically advanced.

Furthermore, the Korean mathematics curriculum seems to emphasize the development of meanings and understanding of fractions, as well as fraction operations with the uses of representations, just as the NCTM publication does. One might notice that in grade 3, different meanings are introduced and developed—first, fractions as parts of a whole, then fractions as parts of a set (see Table 1). In grade 4 the meanings of fractions as ratio and as quotient are addressed.

According to *Commentary on the Course of Study: Elementary School Mathematics* (Korean Ministry of Education 2001), a supplementary document published for the purpose of providing a more detailed discussion and explanation of the course of study, fractions consist of five subconstructs:

1. Part-whole
2. Measure
3. Quotient
4. Ratio
5. Operator

Table 2 provides a brief sequence of the development of different meanings of fractions in the Korean curriculum. The series of fraction meanings in the table are actualized in the classroom beginning in grade 3, with fractions as parts of a whole, through grade 6, with fractions as operators. In analyzing the Korean mathematics curriculum and textbooks, Jeong (2006) reports that the meaning of fractions as operators is first addressed in grade 3 and then again in grades 5 and 6 as multiplication of fractions is introduced and developed. The Korean 7th Curriculum seems to develop these five meanings of fractions across grade levels.

**Textbook organization**

Specifically, then, how are Korean national textbooks organized to introduce and develop fraction concepts? Textbooks reflect how the intention of curriculum is embedded in classrooms, as *potentially implemented curriculum* (Schmidt, Mcknight, and Raizen 1997). In accordance with the Korean mathematics curriculum, only one textbook series has been developed by the Ministry of Education and used nationally for all students at the elementary school level, grades 1–6.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Fraction meaning</th>
<th>Description (e.g., 3/4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Part-whole</td>
<td>Fractions as parts of a whole in continuous models; a whole is sliced into n congruent slices, each slice is encoded as 1/n; 3/4 represents the relative size of parts compared to the whole.</td>
</tr>
<tr>
<td></td>
<td>Part-whole/Operator</td>
<td>Fractions as parts of a set in a discrete model; for example, 1/4 of 12 candies</td>
</tr>
<tr>
<td></td>
<td>Measure</td>
<td>Representation of measured quantities; 3/4 represents the absolute size, such as the measure 3 of the size 1/4 m when 1 m is equally divided</td>
</tr>
<tr>
<td>4</td>
<td>Ratio</td>
<td>A statement about a numerical relationship between two quantities that may or may not involve different kinds of materials; a ratio of 3 to 4</td>
</tr>
<tr>
<td></td>
<td>Quotient</td>
<td>As a result of division between two whole numbers; refers to the fact that the fraction bar / can be a signal to divide; 3/4 = 3 ÷ 4</td>
</tr>
<tr>
<td>5–6</td>
<td>Operator</td>
<td>Similar to the stretching or shrinking process, defines the multiplicative structure of rational numbers: fractions of fractions; 3/4 is 1/4 of 3</td>
</tr>
</tbody>
</table>

Table 2 provides a brief sequence of the development of different meanings of fractions in the Korean curriculum. The series of fraction meanings in the table are actualized in the classroom beginning in grade 3, with fractions as parts of a whole, through grade 6, with fractions as operators. In analyzing the Korean mathematics curriculum and textbooks, Jeong (2006) reports that the meaning of fractions as operators is first addressed in grade 3 and then again in grades 5 and 6 as multiplication of fractions is introduced and developed. The Korean 7th Curriculum seems to develop these five meanings of fractions across grade levels.
Fractions in Korean textbooks

Fraction problems presented in the first lesson of grade 3 (Korean Ministry of Education 2002a, pp. 86–87) show that the fraction concept is introduced in the context of finding parts of a whole (see table 3). The first goal in these activities is to help students construct the idea of equal-share. Students spend two lessons working on equal-share activities with area models. On the basis of this understanding, the third and fourth lessons introduce the fraction as a number to express fractional parts of the whole—the parts that result when the whole, or unit, has been partitioned into equal-sized portions. For example, to indicate equal shares in activity 2, teachers say, “The whole is cut into four parts, and all the parts are the same size; we express one equal share as 1/4 and call it one-fourth.” Both unit fractions and nonunit fractions (e.g., 3/4) are introduced. Only area models such as rectangles, apples, and circles are used for the part-whole relationship.

After establishing the meaning of fractions to represent parts of a whole, the use of fractions as determining parts of a group or set is developed with discrete models, such as candy or counters, in the second semester of grade 3.

<table>
<thead>
<tr>
<th>Korean textbooks introduce fraction concepts in the context of finding parts of a whole to help students construct the idea of equal sharing. Below are introductory problems for fractions presented in the first lesson in grade 3.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learn from real life.</strong> Suppose that you and a friend have one apple to share equally between you. Find out how much each of you will get.</td>
</tr>
</tbody>
</table>
| **Activity 1.** Cut the apple equally in half.  
  • Compare whether the divided parts are the same.  
  • Are they the same? |
| **Activity 2.** Cut the apple in four equal parts.  
  • First cut the apple in half and then divide the halves again evenly.  
  • Compare the results.  
  • Are they the same? |
| **Activity 3.** Evenly divide the rectangular colored paper.  
  • Fold the paper in half.  
  • Cut the paper based on the folded line.  
  • Overlay the two pieces of paper and compare them.  
  • Are they divided equally?  
  • Why do you think so? |

Adapted from Korean Ministry of Education 2002a, pp. 86–87
The part-of-a-group meaning involves selecting objects from a group. The first activity presented in the textbook is to find one-third of six in a real-life situation, such as the following (Korean Ministry of Education 2002b, p. 72):

Six bread buns are to be shared equally among three people. How many buns does each person get?

a. Separate the 6 buns equally into 3 groups.
b. How many buns represent 1/3 of 6?
c. Why do you think this is so?

After students work on several activities where they determine parts of a set, the definition of nonunit fractions is emphasized as a collection of unit fractions (e.g., 3/4 is the same as 3 × 1/4, or adding 1/4 three times). In addition to area models, linear models, such as fraction bars, are addressed and integrated into this activity. Understanding is then extended to comparing fractions with like denominators, using mainly fraction bars as models. The final subunit of grade 3 introduces decimal numbers in connection with fractions. Measurement contexts—such as finding out how much 1 millimeter of 1 centimeter is on a ruler—which lead to such expressions as 1/10 cm and 0.1 cm—are used in grade 3 curriculum (see fig. 1). I noticed only two activities with measurement contexts.

**Unique features**

The previous description of the introduction and development of fractions in grade 3 reveals some unique features about the way in which fractions are taught in Korea. The singular quality of Korean textbooks as opposed to their American or Japanese counterparts is apparent in two respects. First, although elementary school mathematics textbooks in the United States are often criticized because the part-whole meaning is developed mainly with such area models as pies or pizzas, such meaning is developed in Korean mathematics textbooks not only with area models but also with discrete models and measurement models (mainly fraction bars). Second, although Watanabe (2006)—highlighting the prevalence of linear measurement contexts (e.g., liters, number lines) in Japanese mathematics textbooks—suggests the use of measurement as an introductory context for fractions in U.S. textbooks, the Korean mathematics textbooks favor partitioning contexts, delaying measurement contexts until students have established the meanings and uses of fractions as parts of a whole and as a set of objects.

As described earlier, Korean mathematics textbooks introduce fractions mainly with area models in the first semester of grade 3 as numbers to represent the fractional parts of a whole that is divided into equal pieces. In the second semester, the meaning is extended to representing parts of objects. Students are typically asked to find fractional parts of whole numbers, such as 2/3 of 8 objects. In such problem situations, the meaning of a fraction...
is still the fractional parts of a whole that is divided into equal pieces—in this case, a set. But at the same time, the meaning of the fraction is as an operator—a set of instructions or actions to take with regard to another object. A fraction as an operator typically implies partitioning followed by iterating: for example, 2/3 as operator (2/3 of 8) implies first partitioning the object into three equal parts and then making two copies of (iterating) one of those parts. Discrete models are used to develop this meaning. Later, measurement contexts, most prevalent in Japanese textbooks (Watanabe 2006), are used sparingly when introducing decimal numbers in connection with fractions. In measurement contexts, the meaning of fractions is extended to a measure as points or distances on the number line or other linear measurement models (such as a fraction bar). Therefore, in Korean textbooks, fractions are usually introduced and discussed as three different meanings with three different models, but mostly in partitioning contexts.

The primary reason for the Korean textbooks’ emphasis on partitioning contexts (and a fraction as a part-whole relationship) is a familiarity of this context or meaning in students’ real lives, as the teaching manual states:

In real-life situations, there are lots of cases that we share objects in two equal pieces, three equal pieces, and four equal pieces, for example, sharing apples and bread with four friends equally, or sharing pizzas with six friends equally. (Korean Ministry of Education 2002c, p. 203)

Yet a unique quality of Korean textbooks is their balanced use of such models as area, discrete, and measurement models, which are closely related to the development of three different meanings of fractions. Although the most frequent models used are area models, discrete models are subsequently integrated as a way to represent a fraction as a set of objects. Later, measurement models are incorporated mainly with a fraction bar in defining nonunit fractions as a collection of unit fractions (e.g., 3/4 is 3 × 1/4) and in comparing fractions with like denominators. Number lines are included occasionally to represent decimal numbers. This tendency differs completely from the presentation of models in Japanese textbooks, which rarely employ area models, lack discrete models entirely, and use measurement models (mainly number lines) most frequently.

Although the aforementioned features of Korean textbooks are noticeable in contrast to their American or Japanese counterparts, the following qualities are not found or reported in either American or Japanese textbooks. First, considering the challenges students have that are attributed to partitioning contexts or the part-whole approach, Korean textbooks provide an alternative approach that seems to help students deepen their understanding of fractions. One common error that students make in expressing the fractional parts of a whole is to compare fractional parts with the remaining parts of the whole instead of with the whole itself. For example, in the problem in figure 2a, students tend to answer, “One-third,” because one of the four equal parts, or areas, is colored and the remaining three parts are not.

In contrast, Korean textbooks pose the question by showing fractional parts separately
from a whole (see fig. 2b). Once students establish the meaning of fractions in the part-whole contexts, the textbooks address the conventional questions as well.

Another noticeable feature is reflected in the different problem types presented in Korean mathematics textbooks: Each lesson consists of four or five activities. The first activity, called “Learn from real life,” introduces students to a new concept in relationship to a real-life scenario (see table 3). The second and third activities apply the concept. Finally, one or two activities are given for practice. Each activity contains two to five questions requiring different types of answers, which function as teacher questioning to guide students to solve the given activity step by step, even without teachers present. In most cases, such a question as, Why do you think that is true? or Why do you think that is the case? at the end of the activity requires students to justify their solution or method and allows them to share their various strategies.

These types of problems—in particular, problems aimed at facilitating students’ independent thinking—are rarely seen in Japanese textbooks or elementary school mathematics textbooks in the United States. Watanabe (2002) reports that each lesson in Japanese textbooks typically is organized around or consists (solely) of one or two big problems. Unlike elementary school mathematics problems in U.S. textbooks, Korean textbooks provide various leading questions that teachers might use to help students work on an activity problem step by step (or that students might use independently).

Questions to ponder
So, what do these features tell us about the way that Korean elementary school math
books introduce and develop fraction concepts? Some questions arise when we consider both Watanabe’s suggestions (2001, 2006) and the unique features of Korean mathematics textbooks.

Advantages and disadvantages
First, given the different sequencing of contexts and models previously described, a fundamental question is whether one sequence has any potential advantages over another. A unique feature of Korean textbooks is their explicit use of part-whole interpretation to introduce fractions. As described before, the meaning of fractions is introduced as a number to represent the parts from a whole that is equally divided into pieces. The concept is then extended with the use of discrete models to the meanings and uses of a fraction as a set of objects. Later, the meaning of a fraction as a measure is developed in measurement contexts.

Although Watanabe suggests measurement contexts with measurement models (e.g., number lines) as a starting point for learning and teaching fractions, Korean math textbooks delay these contexts and models until students have established the meanings and uses of fractions as parts of a whole and as sets of objects. The choice of the partitioning contexts and the type of models used seems to be influenced by such part-whole interpretation. Accordingly, what are the advantages of measurement contexts with measurement models at the introductory stage? What are the potential limitations of introducing the meaning of fractions as a part-whole interpretation first and then later as a measure?

Type and sequence
Another fundamental question to consider is how to appropriately choose the type of models (and problems) first and in what order those should be used to help students learn. Substantial evidence suggests that using models in fraction tasks is important. For instance, van de Walle recommends that area models, measurement models, and set models be used at all grade levels to develop fraction concepts adequately because “a change in the model usually marks a significant change in the activity from the viewpoint of the children” (2001, p. 209).

So what can be done to help students make better connections among various models when learning fractions? What type of problem best serves each model? Indeed, in addition to more balanced use of three different models, another unique feature in the Korean approach is the presence of different problem types, in activities in particular, functioning as teacher questions that guide students through each activity. What are the advantages and limitations of organizing activities and problems this way in math textbooks? What do we gain by including teacher questions in connection with activities in students’ textbooks?

The best time
The final question to consider is when to introduce fraction concepts. As I pointed out earlier, Korean students consistently perform at or near the top in a variety of international studies not only on fraction-related items but also on many other topics. Instead of delaying formal instruction of fractions in the United States, as Watanabe suggests, the Korean approach shows that introducing fractions earlier than is typically done in North America is a more focused, more balanced approach. Additional questions to consider are whether introductory timing matters, what makes Korean students do better despite an earlier introduction of fraction concepts, and why the U.S. educational system delays introducing fractions. What challenges do students face when fractions are introduced later? What advantages does such a delay present? Many comparative studies of textbook analysis show the advantages of early introduction of mathematical concepts and more balanced use of various types of problems (e.g., Fuson, Stigler, and Barotsch 1988). Along the same line of these studies, the unique features of Korean math textbooks seem to provide some suggestions.

Concluding comments
Korean national curriculum and textbooks have unique qualities that raise pertinent questions about how fractions might be taught in the United States. Rather than providing answers directly, I leave it to the Teaching Children Mathematics audience to make this reading a productive activity.
and a source of possible learning. Understanding how other countries introduce and develop instruction on fractions can offer us guidance:

- Reflect critically on our current practices.
- Improve the quality of both our curriculum materials and instruction.
- Engage in conversations with mathematicians and educators from around the world about what content is most worth teaching, when is the best time to introduce content, and how one might teach it, particularly when teaching fractions.

REFERENCES


Ji-Won Son, sonjiwon@utk.edu, is an assistant professor of mathematics education at the University of Tennessee in Knoxville. Her areas of research include mathematics textbook analysis, elementary and secondary preservice teachers’ knowledge development for teaching, in-service teachers’ curriculum material use, and comparative study.